A Novel Cooperative NOMA Coding for Uplink MTC Networks

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In this paper we consider machine-type communications (MTC) in 5G cellular networks for internet-of-thing (IoT) applications, where a large number of MTC devices simultaneously communicate with a base station (BS) over an uplink channel. To overcome the resource-constrained nature of the MTC nodes, we introduce a novel cooperative non-orthogonal multiple access (NOMA) coding scheme for supporting multi-user uplink transmissions with low complexity and signaling overhead, where the assumption of no feedback link hence no channel state information (CSI) available at the transmitting MTC nodes is taken. To gain deep understanding of its performance, we derive close-form analytical expressions on outage probability and diversity-multiplexing tradeoff (DMT) curve in the asymptotic regime of high signal-to-noise ratio (SNR), and optimize the end-to-end performance in terms of the exponential decay rate of expected distortion (ED). The asymptotic analytical result can be translated into a significant SNR gains of practical implementations with general SNR values, which is further confirmed via extensive simulation.

Index Terms—Cooperative networks, diversity multiplexing trade off, expected distortion, internet of things, machine type communications, non-orthogonal multiple access code.

I. INTRODUCTION

The massive connectivity in the paradigm of internet of things (IoT) for a wide range of emerging applications, such as smart cities, smart grids, vehicular communications (V2X), health and environmental measuring and monitoring, imposes great challenges in 5G mobile systems [1]. Several services, standards, and technologies are envisioned to foster the IoT operations, including massive machine type communications (mMTC) of 5G new radio (5G-NR), narrow-band IoT (NB-IoT), and LTE-M [2]. The design of the IoT architectures needs to align with certain requirements on the sensors/data collecting devices and networks. The requirements on the MTC devices include low cost, low complexity, low signaling overhead, low latency, and low power and computation resources [3].

In generic IoT scenarios, massive number of IoT devices located in a cell attempt to communicate with the base station (BS), and the transmissions are sporadic with low data rates and small data payloads. To schedule the data transmissions of the IoT devices, the BS may employ a centralized-managed access protocol. Due to its high signaling overhead mostly caused by handshaking and multi-step handover process, the centralized access protocols are more applicable to the scenario of smaller numbers of devices each with larger bandwidth demand, and are inefficient for mMTC communications with a large number of IoT devices of small granularity of capacity demand [3].

Grant-free access based approaches have been reported as a more advanced candidate for MTC networks [4], in which each device initiates its transmission including the preamble and data sequence without waiting for any permission from the BS. As a type of grant-free access based approach, non-orthogonal multiple access (NOMA) [5] has been investigated as an enabling technique for supporting the massive access of MTC networks. With grant-free NOMA, messages from several IoT devices can be multiplexed over a common resource block and decoded via successive interference cancellation (SIC) at the BS.

A general method for realizing the grant-free access is by way of a massive multiple access coding, which can be formulated in the information theoretic perspective [6]. The nature of sporadic transmissions by the MTC devices in massive networks, where only a small fraction of devices are active in each transmission slot, enables the deployment of sparse signal processing techniques to detect the active users and decode the messages. This process has been formulated as a sparse optimization problem which can be solved by non-linear compress-sensing (CS) techniques [7]. The literature on CS coding for MTC networks is surveyed in [8]. The CS based grant-free NOMA code [9] which can jointly identify the active users and decode their messages is recently introduced for MTC networks [10] and [11]. Non cooperative joint power and subcarrier allocation (JPSCA) problem for MTC devices is studied in [12]-[14], where non-cooperative game-theory based optimization [15] is deployed for resource allocation.

Recently, machine learning and neural network based optimization techniques have been applied for MTC nodes [16]-[18]. Network secrecy capacity is studied in [12] and [19], and mobile edge computing (MEC) nodes are incorporated to offload the computation from the resource-constraint nodes [19][20]. As MTC nodes are power limited, energy harvesting is proposed for users [21]-[23].

Cooperative schemes where users assist each other in transmission to the BS node, has gained attention from the research community to further enhance the rate, range and latency performance of the MTC uplink networks [24]-[33]. Cooperative uplink NOMA coding has been investigated in terms of reliability, range and sum rate in the literature. [24] has proposed a coordinated direct and relay transmission, and
analyzed the sum rate analysis for both perfect and imperfect SIC decoding, in which some primary users have direct links to the BS while the secondary users are connected through a relay node via a DF relaying scheme. A similar scenario with direct link available for both of the users is studied in [25]. A dedicated relay node deploying a DF relaying protocol allows the users closer to the BS to cooperatively facilitate the transmissions from the far users, where the Ergodic rates of both users are obtained [26]. Unlike previous studies, [28] introduced a full-duplex DF relaying scheme where close-form expressions for sum rate and outage probability are derived. [29] has proposed a relay assisted network deploying amplify-forward (AF) scheme which does not rely on channel disparity among the paired users and prove to achieve the diversity order of two for each user pair.

II. OVERVIEW OF THE PAPER

In this paper we introduce a novel low-complexity layered cooperative NOMA-based access scheme for uplink cooperative MTC networks, where the MTC devices cooperate with each other in transmission of their messages to the BS. The MTC devices deploying the proposed NOMA-based uplink scheme can autonomously adapt their transmissions to the channel condition with no feedback link from the BS, thereby minimizing the signaling overhead and latency, and thus achieving the design of low cost and low power consumption. Specifically, the proposed scheme employs NOMA superposition codes at the MTC devices and SIC based decoding at the BS. With non-orthogonal AF (NAF) [34], each MTC device forwards the messages of other MTC devices via NAF relaying. Further, we consider a joint source-channel coding (JSCC) design in which a layered successive refinement (SR) [36] source code is matched to the proposed multi-layer cooperative NOMA channel code.

We analytically derive the expressions of end-to-end expected distortion (ED) [1] and exponential decay rate of the ED under the high SNR regime, termed distortion exponent (DE) [37]. Note that the DE is a common performance metric in end-to-end distortion systems widely used in evaluating the source-channel coding setup as in [38]. Although the analysis is valid with asymptotically high SNR and sufficiently long source/channel codes, it can provide valid performance upper bound for any practical adaptive coding scheme of cooperative NOMA codes for MTC communications over uplink 5G networks.

The main contributions of this paper are summarized as follows.

- We develop a novel layered NAF cooperative NOMA-based access scheme for uplink MTC communications which enables each MTC device to match its transmission rate to the corresponding channel condition without CSI.

- We propose a maximum likelihood (ML) joint multi-user detection (MUD) scheme matched to SIC decoder, which decodes the messages of the devices and successively decode the layers of each node.

- We explicitly derive the diversity-multiplexing tradeoff (DMT) curve [35] for symmetric multi-layer NAF cooperative NOMA networks and demonstrate its capability of achieving the upper bound of multi-input single-output (MISO) system, whereby the optimal performance under high SNR is proved.

- The proposed uplink channel code is matched to the multi-resolution SR source code, where the end to end DE performance in high SNR regime is fully characterized.

- We verify the proposed cooperative code and prove its superiority via extensive simulation by comparing it with a number of counterparts, including OMA, non-cooperative NOMA code, and AF and DF based cooperative schemes for MTC networks.

The rest of the paper is organized as follows. Section II provides the system model for the considered uplink single cell MTC network and the preliminaries on high SNR analysis of DMT. Section III introduces the proposed multi-layer NAF cooperative NOMA code, and Section IV presents the DMT gain and DE analysis/optimization. Simulation results are presented and discussed in Section V. Section VI concludes the paper.

III. SYSTEM MODEL

A. Multi-User Single Cell Cooperative MTC Uplink Network

We consider the scenario of uplink transmission in a single-cell cellular network as shown in Fig. 1. Let a number of $B$ MTC nodes be in the cell willing to send messages to the BS node over the uplink channel. We group the MTC nodes into $C$ clusters each with $L = B/C$ nodes. Considering the NB-IoT protocol, let the total bandwidth in the cell for each time slot be divided into 48 subcarriers, each being allocated to each cluster in the cell. Let each MTC node be with a single antenna. A NOMA coding scheme is deployed in each cluster, where $L$ MTC nodes in each cluster share the whole available resource block in the allocated subcarrier. Node $i$ is in $[1 : L]$ intending to transmit its message $m_i$ to the BS, termed as node $L + 1$. The MTC nodes in each cluster are assumed to be able to cooperate with each other in transmission of their messages in a half duplex mode. The message of node $i$, $m_i \in [1 : 2^{N R_i}]$, where $N$ is the channel code length and $R_i$ is the coding rate in bits/s/Hz, is encoded into a message sequence $x_i = (x_i(n = 1), x_i(n = 2), ..., x_i(n = N))$, where $x_i$ is transmitted over the network by node $i \in [1 : L]$. In addition to transmission of its own message $m_i$ encoded into sequence $x_i$, each MTC node cooperates in transmission of other MTC node’s messages $m_j$, $j \in [1 : L], j \neq i$, by using the encoding rule

$$ (x_i, y_i) \rightarrow t_i, \quad (1) $$

where $y_i$, defined in (3) is the received vector of length $N$ symbols at node $i$, and $t_i$ is the transmit vector with length $N$ at MTC node $i$. The average energy available for transmission of a symbol at node $i$ is given by

$$ \frac{1}{N} \sum_{n=1}^{N} E[t_i^2(n)] = P, \quad (2) $$
σ is the ratio of average energy of a symbol to the variance of the mean and unitary covariance. The SNR, channel gain for the link between each pair of nodes is given as a sequence received at the BS, is only available at the BS node and no feedback link is constant over a channel block length random variable with covariance equal to one, remaining among the MTC nodes in the cluster with the encoding rule of the transmission of source sequence by the encoder is given by

\[
\mathbf{y}_i = \sum_{j=1 \atop j \neq i}^L g_{i,j} \mathbf{t}_i + \mathbf{z}_i, \quad i \in [1 : L + 1],
\]

where \( \mathbf{z}_i, i \in [1 : L + 1] \) are vectors of additive white Gaussian noise of length \( N \) whose i.i.d. elements have zero mean and unitary covariance. The SNR, \( \rho \) is defined as the ratio of average energy of a symbol to the variance of the noise observed at the receiving end. By setting \( \sigma_z^2 \) equal to 1, \( \rho = \frac{\sigma_t^2}{\sigma_z^2} = P \). Due to the signaling overhead and complexity of requirements of an MTC network, we assume the CSI is only available at the BS node and no feedback link is provided to convey such information to the MTC nodes. The sequence received at the BS, \( \mathbf{Y}_{L+1} \), is decoded using an SIC decoding rule that successively decodes and obtains messages \( (\hat{m}_1, ... , \hat{m}_L) \) of the \( L \) users.

### B. End-to-End Source-Channel Coding

We assume each MTC node is generating/collecting uncompressed data, where the source sequence at each node is modeled as discrete-time continuous-amplitude data sequence \( s^K_i, i \in [1 : L] \) consisting of \( K \) source symbols. We assume the source sequences are zero mean, unit variance with i.i.d. Gaussian distribution.

The source sequence \( s^K_i \) are mapped into the message \( m_i \) by source encoder \( m_i(s^K_i) \), which is subsequently encoded by the channel encoder \( x_i(m_i) \) and sent over the cooperative uplink channel by the encoder \( t_i(x_i, y_i) \) as described above. The rate of the transmission of source sequence \( s^K_i \) is given by \( bR_i \), where \( b = \frac{L}{K} \) is also referred to as bandwidth ratio. At the BS node, the joint decoder assigns estimate \( (\hat{m}_1, \hat{m}_2, ..., \hat{m}_L) \) by jointly decoding the messages, and then the source decoder reconstructs the source sequences according to the estimation \( (\hat{s}^K_1, \hat{s}^K_2, ..., \hat{s}^K_L) \).

A distortion measure \( d \) is defined to characterize the closeness of the source sequence estimations \( \hat{s}^K_i \), reconstructed at the BS, to the original sequences \( s^K_i \) at the MTC nodes. For the transmission of a Gaussian source over the Gaussian network, we assume the squared (quadratic) error distortion measure,

\[
d(s, \hat{s}) = (s - \hat{s})^2.
\]

The average ED over \( K \) symbols is defined,

\[
d(s^K_i, \hat{s}^K_i) = \frac{1}{K} \sum_{t=1}^K d(s^K_i(t), \hat{s}^K_i(t)),
\]

and the expected distortion for user \( i \) is given by

\[
D_i = E[d(s^K_i, \hat{s}^K_i)],
\]

which is the ED averaged over all the statistical random variables of the network, including random source sequences, Additive white Gaussian noise (AWGN) at the nodes and channel fading gains. Thus, the overall network ED is defined by

\[
D = \max_{i \in [1 : L]} D_i.
\]

We assume \( K \) is large enough to achieve the rate-distortion function of the source. The rate distortion function for the Gaussian source, with average power 1, is given by

\[
R_S(D) = \inf_{\rho(\hat{s}|s), E[(\hat{s}-s)^2] \leq D} I(S; \hat{S}) = \frac{1}{2} \log \left( \frac{1}{D} \right).
\]

The distortion of reconstruction of source sequence \( s^K_i \) by \( \hat{s}^K_i \) at the BS is given by the inverse function of \( R_S(D) \) with

\[
D_i = 2^{-bR_i},
\]

where \( R_i \) is the channel code rate for node \( i \). We intend to optimize the ED of transmission of source sequence at each MTC node of \( L \) cooperating nodes in a cluster over the uplink transmission, by devising the cooperative scheme \( t_i, i \in [1 : L] \) and proper rate and power allocation to each node. The problem is formulated as follows

\[
\min_{\{t_i\}_{i=1}^L, \{R_i\}_{i=1}^L} \{D\}
\]

\[
s.t. E[T_i^2] \leq \rho_i,
\]

where \( D \) is defined in (7) and with the assumption that CSI is only available at the BS and no feedback link to the MTC nodes and no centralized resource allocation by the BS node.

Since this is a non-linear non-convex problem and is not tractable in general case, we reformulate the problem in high SNR regime where \( \rho \rightarrow \infty \), and consider the symmetric case, where the rate and power constraint of the nodes are assumed to be the same. Hence we have \( (R_i, \rho_i) = (R, \rho), \forall i \in [1 : L] \). With this, we will introduce a novel suboptimal solution based on our NOMA coding where the source sequences at each node are encoded into refining layers. We will prove the proposed NAF relaying based NOMA cooperative scheme to be optimal in a symmetric network with high SNR.
C. High SNR Approximation, DMT and DE Analysis

Considering a slow fading network, the mutual information \( I_i \), corresponding to the transmission rate of MTC node \( i \) to the BS, is a random variable, as it is a function of the random fading gain and other channel uncertainties. Defining the channel fading gain vector \( g = \{g_{i,j}, i \in [1 : L + 1], j \in [1 : L]\} \), the outage event \( O_i \) for a fixed channel code rate \( R_i \) is defined as \( O_i = \{g | I_i < R_i\} \), and the outage probability is \( P_{O_i} = Pr[g | I_i < R_i] \). In the high SNR regime, defining the multiplexing gain, \( r_i \) as \( R_i = r_i \log \rho \), the diversity gain for node \( i \) is defined as

\[
d(r_i) = \lim_{\rho \to \infty} -\frac{\log P_{O_i}(r_i)}{\log \rho},
\]

(10)

and the curve \( d(r_i) \) is the DMT curve for the channel.

Due to the high SNR, the channel coding performance is captured by \( d(r_i) \) and the DE defined as,

\[
\Delta_i = \lim_{\rho \to \infty} -\frac{\log D_i}{\log \rho},
\]

(11)

fully characterizes the end to end performance of the source and channel coding scheme. For the problem of source sequence transmission over the cooperative uplink network, the overall network DE is defined as

\[
\Delta = \min_{i \in [1 : L]} \Delta_i
\]

(12)

With the symmetric case, we have \( r_i = r, \forall i \in [1 : L] \) and the end to end performance optimization problem given in (10) under the high SNR regime can be reformulated as

\[
\max \left\{ \Delta \right\}
\]

s.t. \( E[T_i^2] \leq \rho_i \).

IV. PROPOSED LAYERED NAF COOPERATIVE NOMA CODE

A. Layered Structure

With the proposed layered NAF cooperative NOMA code, a successive refined (SR) layered source code structure is employed where the Gaussian source sequence \( s^K_i \) at MTC node \( i \) is encoded into \( M \) refining descriptions \( \{m_{1,i}, ..., m_{M,i}\} \). Considering the SR property of the Gaussian source [36] under a common distortion measure \( d \), there is no loss of optimality by successively describing layers of the source. Hence the following \( M \) rate-distortion functions are simultaneously achievable,

\[
\sum_{j=1}^{k} bR_{j,i} = R_S(D_{k,i}), k \in [1 : M],
\]

(13)

for each source sequence, where \( R_S(D) \) is the rate-distortion function of the underlying source code defined in (8).

At each MTC node \( i \), the sequence \( s^K_i, i \in [1 : L] \) is compressed into \( M \) layers, and channel-encoded into a superimposed sequence. At the receiver side, the BS node decodes for layers \( j \in [1 : M] \) for each source node \( i \in [1 : L] \). Based on the successfully decoded layers up to layer \( j \) for each source message, the source sequence \( s^K_i, i \in [1 : L] \) is reconstructed at the destination by \( \hat{s}^K_i(m_{1,i}, ..., m_{j,i}) \).

At MTC node \( i \), the \( M \) successively refining layers of the source sequence are separately channel-encoded and are superimposed. For each source sequence we have \( M \) channel sequences of rates \( \{R_{1,i}, R_{2,i}, ..., R_{M,i}\} \), where \( R_i = \sum_{j=1}^{M} R_{j,i} \). The \( M \)-layer channel sequence at node \( i \) is given by \( \{x_{1,i}(m_{1,i}), x_{2,i}(m_{2,i}), ..., x_{M,i}(m_{M,i})\} \).

At the MTC node \( i \), channel sequence \( x_i^N \) is constructed by superposition of the \( M \) layers,

\[
x_i(m_{1,i}, ..., m_{M,i}) = \sum_{j=1}^{M} \sqrt{\rho_{j,i}} x_{j,i}(m_{j,i}),
\]

(14)

where \( \rho_{j,i}, j \in [1 : M] \) defines the allocated power to layer \( j \) of transmitter \( i \) and \( \sum_{j=1}^{M} \rho_{j,i} = \rho \).

The BS node decodes the received messages

\[
(m_{1,i}, m_{2,i}, ..., m_{M,i})
\]

(15)

of source sequence \( s^K_i, i \in [1 : L] \) and recovers as many layers as possible based on the instantaneous fading status, channel realization, and additive noise level. Based on the number of successfully decoded layers, \( \hat{s}^K_i \) is reconstructed accordingly. In the case that \( k \) layers are decoded for source \( i \), the distortion rate functions is given by,

\[
D_{k,i} = 2^{-b \sum_{j=1}^{k} R_{k,i}}, k \in [1 : M].
\]

(16)

The outage event for layer \( k \) is denoted by

\[
O_{k,i} = \{g : I_{k,i} < R_{k,i}\}.
\]

(17)

where \( I_{k,i}, k \in [1 : M] \) is the mutual information of the \( k \)th layer of node \( i \), assuming the lower layers are decoded by the SIC decoder and the higher layers are interference terms. In the case of layered coding, the ED of node \( i \) is defined as

\[
D_{i} = \sum_{j=0}^{M} (P_{O_{j+1}} - P_{O_j})D_{j,i},
\]

(18)
where $D_{i,t}$ is given by (16) and $P_{O_{M+1}} = 1 - \epsilon$, $P_{O_{0}} = 0$, $D_{0,t} = 1$, $\epsilon > 0$.

With the high SNR assumption, $\rho \rightarrow +\infty$, the layered network overall ED ($D^L$) and distortion exponent ($\Delta^L$) are given in (7) and (12), respectively, where $D_i$ is replaced by $D^L_i$ as given in (18). Thus, the distortion exponent optimization problem is given as follows,

$$\min_{\{x_i\}^{M}_{i=1}} \{\Delta^L\}$$

s.t. $E[T^2_1] \leq \rho_i$.

B. NAF Cooperative uplink NOMA Code

The proposed NAF based cooperative NOMA scheme for the uplink MTC network which is matched to the layered coding scheme described above, is outlined as follows. We characterize the cooperative encoding functions $t_i(x_i, y_i)$ by which the cooperation scheme is defined.

Let a cooperation frame consist of $L$ transmission blocks. In transmission block $i$, only node $i$ is active and it transmits its own layered messages $(m_{ij})^M_{i=1}$ encoded into $x_i$, and besides it assists in transmission of another node $j$. Hence the message sequences in cooperation frame $p$ is denoted as $x^1_i, x^2_i, \ldots, x^P_i$.

We consider $P$ consecutive cooperation frames form a super frame, during which the message sequences are expressed as

$x^1_i, x^2_i, \ldots, x^1_j, x^2_j, \ldots, x^2_L, \ldots, x^P_i, x^P_j, \ldots, x^P_L$.

In each super frame the cooperating node assignment for each transmitting node is based on the right circular shift of $1, 2, \ldots, L$, where for super frame $l$, the cooperating node $i$ for transmitter $j = 1, \ldots, L, j \neq i$ is assigned by

$$i = j + l \pmod{L} \quad \text{(20)}$$

There are $L - 1$ distinct configurations for the cooperation assignment, so $L - 1$ consecutive super frame are considered, where in each super frame the cooperation node assignment is defined by the rule given in (20), and is fixed during the $P$ cooperation frames of the super frame. This forms the transmission frame of length $LP(L - 1)$ channel code blocks. In each transmission frame, totally $LP(L - 1)$ messages are transmitted to the BS from the $L$ nodes in a cluster, each node contributing $P(L - 1)$ messages.

The cooperating node $i$ at cooperation frame $p$, relays the sequence $x^p_j(m_{ij})$, besides transmitting its own message sequence $x^p_i(m_{i})$ by linearly combining the two sequences. For each super frame $l$, $p'$ is given by

$$p' = \begin{cases} p, & j + l \pmod{L} > j \\ p - 1, & j + l \pmod{L} \leq j. \end{cases} \quad \text{(21)}$$

Hence the encoding function of the cooperative scheme, $t_i(x_i, y_i)$ is defined as the linear combination of the node $i$ message sequence and the received signal at node $i$

$$t^p_i = \alpha_{i,p}x^p_i + \beta_{i,p}y^p_i, i \in [1 : L] \quad \text{(22)}$$

where the coefficients of $\alpha_{i,p}$ and $\beta_{i,p}$ in (22) are set such that the overall power limit of $\rho$ is met by the nodes at each transmission block and $y_{i,p}$, the received signal at source node $i$ in cooperation frame $p$ is defined as

$$y^p_i = g_{i,j}t^p_j + z^p_i, i \in [1 : L]. \quad \text{(23)}$$

The received sequence at the BS in each super frame is given by

$$y^p_{L+1} = g_{L+1,i}t^p_i + z^p_i, i \in [1 : L], p \in [1 : P], \quad \text{(24)}$$

where $t^p_i$ is given in (22) and the super frame structure is shown in Fig. IV-A.

The recursive cooperation relationship (22) and (23), along with the cooperating node assignment by (20), ensure the transmission of each source node to be relayed by the other $L - 1$ nodes in every transmission frame.

The BS node decodes the $P(L - 1)$ messages of each source node sent during the transmission frame, and in total it receives $LP(L - 1)$ messages from all the senders during the transmission frame of length $LP(L - 1)$, leading to an overall rate as $R$. The encoder and decoder for the proposed NAF cooperative NOMA code is given in Fig. IV-A and Fig. ??.

V. HIGH SNR ANALYSIS FOR LAYEDER NAF COOPERATIVE UPLINK NOMA CODE

A. Layered DMT Curve Analysis

In this section we analyze the DMT curve of the proposed layered NAF cooperative NOMA scheme for the uplink MTC network. We provide a lower bound on the achievable diversity and show that it achieves the upper bound of MISO network, hence prove the optimality of the layered scheme. Our approach follows the techniques used in [34], [35] and [38]. Assuming only a subset of source nodes $Q \subset [1 : L]$ are transmitting messages, we consider a suboptimal decoder which picks a partial observation vector consisting of only one observation for source $j \in Q$ in each transmission frame.

For the transmitted sequence $x^p_j, j \in Q$ which is relayed by node $i$ in cooperation frame $p$, the decoder picks the observation based on the following rule

- In case the cooperating node $i \notin Q$, the decoder picks either the transmit sequence of $g_{L+1,j}a_{j,p}x^p_j$ or the relayed sequence $g_{L+1,j}b_{j}g_{i,j}a_{j,p}x^p_j$, which is determined by whether $g_{L+1,j}$ or $g_{L+1,i}$ is the dominant channel.

- In case $i \in Q$, the decoder picks the direct transmission of $g_{d,j}a_{j,p}x^p_j$ over the relayed signal.

The outage event for layer $k$ of the suboptimal SIC decoder can be expressed as

$$O_k = \frac{1}{|Q|P(L - 1)\log|I + \Sigma_{x_k} G^h(G_{x_{k+1}}G^h + \Sigma) - 1|} < r_k \log \rho \quad \text{(25)}$$
where $G$ is square matrix of size $|Q|P(L - 1)$ corresponding to the $|Q|$ observation per cooperation frame over a transmission frame of length $L - 1$ super frames. It is a lower triangular matrix with nonzero elements of $g_{L+1,k}a_{i,j}p^{r_{ij}}$ or $g_{L+1,k}b_{i,j}a_{i,j}p^{r_{ij}}$ based on the decoding rule of the decoder. The matrix $\Sigma_{rk}$ is a diagonal matrix, where the nonzero elements are set such that the transmit signals $t_{ij}^k$ meet the power constraints $E[|t_{ij}^k|^2] \leq \rho$. $\Sigma_{Z}$ is the covariance of the noise vector. With the high SNR assumption, the channel gain exponents $v_i, u_{ij}, i, j \in [1 : L]$ are defined as $|g_{L+1,k}|^2 = \frac{1}{1 + \gamma_i}, |g_{i,j}|^2 = \frac{1}{1 + \gamma_{ij}}, \text{and the channel gain exponent vector } u \text{ is defined, } u = \{(v_i)_{i=1}^L, (u_{ij})_{i,j=1}^L\}$. We define the set $O'$ as the channel exponents set, where

$$O'_k = \left\{ u : \log |I + G\Sigma_{rk}G^h(G\Sigma_{rk+1}G^h + \Sigma_Z)^{-1}| < |Q|P(L - 1)r \log \rho_k \right\}.$$  

The achievable DMT curve of the proposed layered NAF cooperative NOMA code is proved in the following theorem.

**Theorem 1.** The DMT curve of a symmetric layered NAF cooperative uplink NOMA code of $L$ nodes deploying $M$-layer coding is lower bounded by

$$d^L(r_k) = L(1 - r_1 - \ldots - r_k), k \in [1 : M].$$  

**Proof.** We analyze the high SNR outage event for layer $k$ by considering the event $O_k'$ in (26). Since the channel matrix $G$ is a lower triangular matrix, the determinant is the multiplication of the diagonal elements. Based on the decoding rule, in case the cooperating node is in $Q$, the direct source signal is used by the decoder; and in case of $i \notin Q$, the best link of $g_{L+1,i}$ and $g_{L+1,i}g_{i,j}$ is chosen. Also the noise covariance in the high SNR can be shown to be $\Sigma_{Z} \leq I$. Hence, the outage event of layer $k$, $O_k'$, in (26), can be written as

$$O_k' = \left\{ u : \log \left| G\Sigma_{rk}G^h \right| < |Q|P(L - 1)r \log \rho \right\}.$$  

The aggregate power allocated to layers $(k, k + 1, \ldots, M)$, $\sum_{i=k}^{M} \rho_i$ is set to

$$\bar{\rho}_k = \sum_{i=k}^{M} \rho_i = \rho^{\eta_k}.$$  

Replacing the values of channel gains of $G$ by the corresponding values, given by the decoding rule, and further simplification, the term $|G\Sigma_{rk}G^h|$ in (28) in terms of power allocation exponent $\gamma_k$ and channel gain exponents $\{(v_i)_{i=1}^L, (u_{ij})_{i,j=1}^L\}$ is

$$|G\Sigma_{rk}G^h| = \rho^{\eta_k},$$

where $\eta_k = |Q|P(L - 1)\gamma_k - \sum_{j \in Q} \left( (|Q| - 1)Pv_j + \sum_{i \notin Q} \left( \min\{v_j, u_{ij} + v_i\} \right)(P - 1 + v_j) \right)$.

Hence $O_k'$ can be written as

$$O_k' = \left\{ u : \rho^{\eta_k} - \rho^{\eta_k+1+|Q|P(L-1)r_k} < \rho^{|Q|P(L-1)r_k} \right\}.$$  

We set the power allocation coefficients recursively as

$$\gamma_{k+1} = \gamma_k - r_k - \epsilon,$$  

for some $\epsilon > 0$. Hence

$$O_k' = \left\{ u : \rho^{\eta_k} < \rho^{|Q|P(L-1)r_k} \right\}.$$  

Replacing the value for $\eta_{k}$ given in (30), the outage event for layer $k$ is

$$O_k' = \left\{ u : \sum_{j \in Q} v_j > |Q|(1 - r), \sum_{j \in Q} (|Q| - 1)v_j + \sum_{i \notin Q} u_{ij} + v_i > |Q|(L - 1)(1 - r) \right\}.$$  

The outage probability is

$$P_{O_k} = \int_{O_k'} p_u(u)du.$$  

Deploying the typical outage event method of [34, Result 5], the outage probability in the high SNR, $P_{O_k}$, is given by

$$P_{O_k} = \rho^{-d_k(r_k)}$$

where $\rho$ denotes high SNR exponent equality, and $d_k(r_k)$ is given by

$$d_k(r_k) = \inf_{u \in O_k'} \sum_{j=1}^{L} v_j + \sum_{i \notin Q} u_{ij}.$$  

The infimum is achieved by the lower bounds given in (32). Replacing the values of $v_j$ and $u_{ij}$ we have

$$d^L(r_k) = |Q|(1 - \sum_{i=1}^{k} r_i)$$

$$+ (L - 1)(1 - \sum_{i=1}^{k} r_i) - (|Q| - 1)(1 - \sum_{i=1}^{k} r_i) = L(1 - \sum_{i=1}^{k} r_i)$$  

which proves the achievable diversity order given in (27).

So far we obtained the achievable lower bound on the diversity gain of the proposed coding scheme. On the other hand, an upper bound is derived by assuming all the $L$ MTC nodes in the cluster form an antenna array of length $L$ and jointly transmit to the BS, forming a multi-input single output (MISO) system. The layered MISO system diversity gain is given in [38] and is the same as the (27) hence indeed, the achieved curve is the optimal DMT curve for the proposed cooperative uplink NOMA code.
B. End to End Performance Analysis: DE Optimization

Up to now, we have derived the DMT curve of symmetric layered NAF cooperative uplink NOMA system, where the \( M \) layers of the L MTC nodes are assumed to have the same diversity gains \( r_{ij} = r_j, i \in [1: L], j \in [1: M] \). The next step is to solve the achievable average DE of the considered end to end system. The conditions on the optimal multiplexing gain assignment \( \{r_j\}_{j=1}^M \) are firstly given in the following lemma.

**Lemma 1.** The multiplexing gain assignment, \( r_j \) of the layers \([1 : M]\) of the \( L \) MTC nodes in a cluster over a layered NAF cooperative NOMA network, which optimizes the achievable ED in the high SNR regime in (19), is given by

\[
b \sum_{i=k+1}^{M} r_i - d^L(r_k) = 0, k \in [1 : M].
\]

**Proof.** We intend to optimize the ED in (19) by assigning the optimal multiplexing gains for each layer at each MTC node. The optimization problem can be written as

\[
\begin{aligned}
& \min \left( -b \sum_{i=1}^{M} r_i \right) \\
& \text{s.t. } b \sum_{i=1}^{M} r_i \leq d^L(r_k) + b \sum_{i=k+1}^{M} r_i, k \in [0 : M].
\end{aligned}
\]

We form the Lagrangian

\[
L = -b \sum_{i=1}^{M} r_i + \sum_{k=0}^{M} \mu_k \left( b \sum_{i=k+1}^{M} r_i - d^L(r_k) \right)
\]

Considering the complementary slackness condition \([39]\), the optimal solution satisfies the

\[
\begin{aligned}
\nabla L &= 0 \\
\mu_k \left( b \sum_{i=k+1}^{M} r_i - d^L(r_k) \right) &= 0, k \in [0, M] \\
\mu_k &\geq 0, k \in [0, M]
\end{aligned}
\]

solving \( \partial L / \partial r_i = 0 \) for \( i \in [0, M] \), the coefficients \( \mu_k \) are

\[
\mu_{k+1} = \frac{L}{b} \mu_k,
\]

where \( \mu_0 = \frac{b^M}{\sum_{k=0}^{M} b^{M-k-1}} \). Hence for \( b > 0 \), \( \mu_k, k \in [0, M] \) are strictly positive; thus the optimal multiplexing gain vector \( \{r_1, r_2, ..., r_M\} \) satisfies

\[
b \sum_{i=k+1}^{M} r_i - d^L(r_k) = 0, k \in [1 : M]
\]

This completes the proof of Lemma 1.

The following theorem characterizes the DE for a finite number of layers and in the case of asymptotic approximation of an infinite number of layers.

**Theorem 2.** The achievable DE of \( L \) Gaussian sources with unit covariance, each observed at a MTC node \( i \in [1 : L] \) and encoded into \( M \) layers concatenated to a symmetric NAF cooperative uplink NOMA code, is given by

\[
\Delta^M = \frac{bL A_M}{L + bA_M},
\]

where \( A_M \) is given by

\[
A_M = \sum_{i=0}^{M-1} \frac{b}{L^i}.
\]

The optimal DE of the layered NAF cooperative NOMA code in the limit of infinite number of layers, \( M \rightarrow \infty \), is given by

\[
\Delta^+ = \min\{b, L\}.
\]

**Proof.** Considering the layered diversity gain of (27), the ED equation of (18) for layered code, and the optimal diversity gain assignment given in Lemma 1, the optimal multiplexing gain of each layer is computed as

\[
r_k = \left( \frac{b}{L} \right)^{k-1} r_1,
\]

for the last layer we have

\[
r_M = \frac{L}{b + L A_M},
\]

where \( A_M \) is given by (42). The DE can be written as

\[
\Delta^M = b(r_1 + r_2 + ... + r_M)
\]

further simplification for \( \Delta \) gives the equation of (41).

In the case of the infinite number of layers, \( A_M \) simplifies to \( \frac{L}{b} \) and the DE given in (41) is equal to \( b < L < b \), and the DE simplifies to \( L \) for \( b > L \).

This completes the proof.

It can be seen that the layered NAF cooperative NOMA code achieves the MISO upper bound \([38]\) in terms of DE in the asymptotic regime with an infinite number of layers. This implies that the proposed layered code is indeed optimal in the considered network setup.

In order to compare the result of the layered scheme with the single layer case, we formulate the DE of a single layer code over uplink network which is given in the following lemma.

**Lemma 2.** The DE of a single layer NOMA code over an uplink network is given by

\[
\Delta^{SL} = \frac{Lb}{L+b}.
\]

**Proof.** The DMT curve of the single layer uplink network is given in [34]

\[
d(r) = L(1-r).
\]

The ED in terms of outage event and source distortion-rate function for a single layer code is

\[
ED = (1 - P_O)D + P_O.
\]

In the high SNR region, we have

\[
\lim_{\rho \to \infty} ED = \rho^{-br} + \rho^{d(r)},
\]

hence the optimal multiplexing gain is

\[
br = L(1-r).
\]

Therefore, the DE is

\[
\Delta = \frac{bL}{L+b}.
\]

This completes the proof.
VI. SIMULATION RESULTS

In this section we study the performance of the proposed layered cooperative NOMA coding for uplink MTC network through Monte Carlo simulations. We set the simulation parameters according to 3GPP NB-IoT given in table I. We consider a cell with radius 500m where the BS node is located at the center of the cell. Location of the MTC nodes are randomly generated and uniformly distributed within the cell. The available bandwidth is divided into set of 48 subcarriers each with a spacing of 3.75 KHz. The channel gain of each node is modeled by

\[ g_i \propto \frac{d_i^{-\eta}}{d_i^2} \]

where \( g_i \) is the random variable generated based on Rayleigh distribution, \( d_i \) represents the distance of node \( i \) to the BS node, and \( \eta \) is the path loss exponent. Similarly, the channel gain between MTC node \( i \) to \( j \) is set by a Rayleigh fading distribution and path loss gain. We consider AWGN with power spectral density of -173 dBM/Hz, and the max power budget for each node is set to 23 dBm.

\[
\text{TABLE I SIMULATION PARAMETERS}
\begin{array}{|c|c|}
\hline
\text{Simulation parameters} & \text{Value setting} \\
\hline
\text{Cell radius} & 500m \\
\text{Transmission bandwidth} & 180KHz \\
\text{Number of subcarriers} & 48 \\
\text{Subcarrier Bandwidth} & 3.75KHz \\
\text{Noise power spectral density} & -173dBm/Hz \\
\text{Transmission power of the MTC nodes} & 23dBm \\
\text{path loss model} & (38 + 30 \log d_i)dB \\
\text{Fading model} & \text{Rayleigh fading} \\
\hline
\end{array}
\]

For the first set of simulation, 480 MTC nodes are divided into 48 clusters, and the nodes in each cluster share the same resource block and cooperate in transmission of their messages to the BS node. In Fig. 3, the proposed layered cooperative NOMA scheme, named as NAF-NOMA, is compared with some other cooperative schemes available in the literature, including DF based [31] and AF based [32] cooperation schemes, in terms of the average bit error rate (BER) for each MTC node. A generic non cooperative NOMA scheme, namely NC-NOMA, is also implemented as a benchmark.

It is shown that all the NOMA schemes significantly improve the BER performance compared to the non-layered orthogonal scheme due to much enhanced adaptability of their transmissions to the instantaneous channel realization at each MTC node. Besides, the NAF scheme achieves better performance than the others as it provides higher diversity gain.

We further examine the Ergodic sum rate of the users in each cluster that is illustrated in Fig. 4 as a function of the number of MTC nodes per sub carrier. Increasing the number of MTC nodes in each cluster can boost the sum rate per cluster at the expense of increased complexity of the SIC decoder due to a larger number of layers that are interfering with each other. It is shown that the NAF coding outperforms the AF and DF coding schemes as it enjoys a higher multiplexing gain.

Secondly, we examine the end-to-end performance in terms of ED and DE in high SNR. Fig. 5 shows the inference due to the number of layers upon the DE performance. It can be seen that even with a limited number of layers, the DE performance is considerably improved against the non-layered coding case. As the number of layers gets higher the performance tends to the MISO upper bound which confirms the results of Theorem 1.
Finally, we examine the ED performance of the proposed coding in general SNR values as shown in Fig. 6, where the AF and DF NOMA schemes are taken for comparison. The performance of a limited feedback (FB) system where partial knowledge of the channel is sent back to the MTC nodes. We numerically adjust the power allocation for each case of FB systems and provide the results in this graph. Also a genie-aided NOMA scheme where all the nodes have full channel knowledge is given as benchmark. We consider a limited feedback non-layered AF scheme where feedbacks of partial CSI are sent to the transmitting nodes.

It can be seen that the layered NAF-NOMA scheme provides a significant SNR gain compared to the non-layered scheme and improves performance over the layered AF-NOMA and DF-NOMA, respectively. Further, the layered NAF-NOMA scheme can yield similar performance of a limited feedback network, while in the asymptotic case of an infinite number of layers, it matches the case of full CSI at the transmitting nodes, hence mitigating the lack of channel knowledge at the MTC nodes.

VII. CONCLUSION

In this paper a novel layered NAF cooperative NOMA code was proposed for uplink multi-user cellular transmissions, aiming to fit into the scenario of MTC based IoT systems. It is shown that the proposed scheme can achieve the DMT curve of the MISO upper bound without channel feedback, indicating that the optimal curve is fully characterized in the case of symmetric network. Simulation results confirmed that the proposed NAF cooperative NOMA scheme outperforms the AF and DF based coding in terms of sum-rate, BER and ED. The low complexity encoder with low signaling overhead makes it a proper solution for uplink IoT communications over 5G MTC networks.

REFERENCES


