Fuzzy Inference System Modelling of the Mascarenes Anticyclone center Trajectory

Georgines Jacobsen Rasoarinirina\textsuperscript{a*}, Harimino Andriamalala Rajaonarisoa\textsuperscript{b}, I. Parker Ramahazosoa\textsuperscript{c}, Adolphe Andriamanga Ratiarison\textsuperscript{d}

Atmosphere, Climate and Oceans Dynamics Laboratory (DyACO), Physics and Applications, Sciences and Technologies, University of Antananarivo, Madagascar
\textsuperscript{a} georginesjacob@gmail.com, \textsuperscript{b} hariminondriamalala@gmail.com, \textsuperscript{c} ramahazos@yahoo.fr, \textsuperscript{d} adolphe.ratiarison@univ-antananarivo.mg

*Corresponding Author, Georgines Jacobsen Rasoarinirina, georginesjacob@gmail.com

Abstract

This work objective is to determine the parameters of the fuzzy inference system model that best models the Mascarenes anticyclone trajectory. Our study area extends from 20°E to 110°E longitude and from 15°S to 50°S latitude. We start from the atmospheric pressure reanalysis data in grid point to determine the Mascarene anticyclone center. This center is none other than the center of the last closed Anticyclone’s isobar. The monthly climatological-mean value of the center coordinates are the data to be modeled by fuzzy inference system. The considered model parameters are the partitions number of the discourse universe and the model order. After evaluating the deviation between the input data and the simulated data, the minimum deviation is obtained with order model 2 by using 50 numbers of partitions.

Keywords

Artificial intelligence; Modeling, Fuzzy Inference System; Mascarenes anticyclone; Partitions number; Model order.

1. Introduction

Recently, many researchers in meteorology and climatology are interested in the anticyclone and their effects. These studies are often related to decadal variability, influence on atmospheric warming, and space-time climatology [1] [2]. An anticyclone (a depression respectively) is a closed isobaric system which atmospheric pressure value increases (decreases in case of a depression) towards the interior [3] [4]. Traditionally, the middle of high pressure corresponds to the center of the anticyclone [5]. The anticyclone (respectively depression) is generally associated with clear skies (respectively cloudy skies with meteorological phenomena) [6] [7].
The position of the anticyclone center varies according to the observation time [1]. Nevertheless, there are semi-permanent anticyclones in the world such as the Azores anticyclone, Siberian anticyclone, North American anticyclone, St. Helena anticyclone, ... and the Mascarenes anticyclone. Madagascar submits climatological processes influenced by the Mascarene anticyclone [8]. Its center of action moves according to the season in the Indian Ocean [9] [10] [4]. That is why it determines the weather type as well as many other parameters such as the breezes activity, the different distribution types of forests in Madagascar [10]. The anticyclone position is then a key point of various research fields. This is the reason why our work focuses on the trajectory study the Mascarenes anticyclone center.

The trajectory studies always come down to its modeling. Various mathematical models are able to do this modeling. In the framework of our work, we will exploit the Mamdani fuzzy inference system [14].

2. Methods

2.1. Study Area

The study area is located between 20°E to 110°E longitude and 15°S to 50°S latitude, delineated as shown in Figure 1.

![Figure 1: Study area](image)

2.2. The Experimental Data

We manipulate daily atmospheric pressure reanalysis data from January 1st 1979 to December 31th 2018 from ECMWF. The grid point data with a spatial resolution of
2.5° x 2.5° are netcdf data (.nc extension). From these daily atmospheric pressure data (data in Pascal then converted into hectopascal), for each grid point, we calculate the monthly climatological average values. Then, we draw isobars plot. Thus, for each month of the year, we determine the centers coordinates (longitude and latitude) after measuring the diameters of closed isobars.

The input data for the model are the latitude and longitudes values of the anticyclone center for each year month as shown in Table 1.

**Table 1:** Geographic coordinates of the anticyclone Mascarene center in monthly climatological average

<table>
<thead>
<tr>
<th>Month of the year</th>
<th>Longitude (°)</th>
<th>Latitude (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>97</td>
<td>-33</td>
</tr>
<tr>
<td>February</td>
<td>85</td>
<td>-30</td>
</tr>
<tr>
<td>March</td>
<td>71</td>
<td>-28</td>
</tr>
<tr>
<td>April</td>
<td>45</td>
<td>-29.5</td>
</tr>
<tr>
<td>May</td>
<td>42.3</td>
<td>-30</td>
</tr>
<tr>
<td>June</td>
<td>69</td>
<td>-33.5</td>
</tr>
<tr>
<td>July</td>
<td>59</td>
<td>-35</td>
</tr>
<tr>
<td>August</td>
<td>68</td>
<td>-34</td>
</tr>
<tr>
<td>September</td>
<td>61</td>
<td>-33.5</td>
</tr>
<tr>
<td>October</td>
<td>69</td>
<td>-33.5</td>
</tr>
<tr>
<td>November</td>
<td>82.77</td>
<td>-34</td>
</tr>
<tr>
<td>December</td>
<td>88.5</td>
<td>-32</td>
</tr>
</tbody>
</table>


Fuzzy logic is a subfield of artificial intelligence that aims at formalizing approximate situations and simulates human reasoning. It integrates a set of fuzzy rules that allows to establish a relation between the input and output variables. We use this type of model to model the Mascarene anticyclone trajectory.

Fuzzy logic modeling is divided into three main steps:
- Fuzzification;
- Fuzzy inference;
- Defuzzification.

For our model, the model order r equals to the number of model inputs. We put as input the latitudes and longitudes of the rank month: m, m-1, ..., m-r+1. The model outputs are the predicted latitude and longitude for the month m+1.

2.3.1. The Fuzzification

Fuzzification is the operation of converting a real value into a fuzzy value. Before fuzzifying input variable, the modeler defines first the discourse universe of the real variable and then builds the fuzzy sets on this universe. Thus, the number of discourse
universe is equal to the sets of fuzzy numbers built on this universe which is a key model parameter. For example, for the input variable longitude, the range of the longitude data series is [42.3 97]. On this interval, we construct for example three fuzzy sets named A1, A2 and A3.

By definition, a fuzzy set is a set containing elements whose membership degrees vary from [0, 1].

Let U be the discourse universe associated to the real variable x (e.g., longitude), A a subset of U defined by [15]:

$$A = \left\{(x, \mu_A(x)), x \in U \right\}$$

Fuzzy set theory states that if A is a fuzzy set of membership function, then:

$$\forall x \in U, \mu_A(x) \in [0, 1]$$

Let us take the linguistic variable longitude example. We consider three fuzzy sets A1, A2 and A3 of triangular membership function as shown in Figure 2. For a value of longitude 80, the corresponding linguistic variables are A2 and A3 with 25% and 50% respectively. In our work, we retain only the linguistic variable with the highest degree of membership that is A2.

In the following, we adopt the following notation $F(80) = A2$

![Figure 2.a: Example of longitude fuzzification](image-url)
2.3.2. The Fuzzy Inference

Fuzzy inference comes after the fuzzification process. It consists in applying the fuzzy rules to the input linguistic variables to have a fuzzy set of output. A fuzzy rule has the format IF "Conditions" THEN "consequence".

The conditions refer to the linguistic variables after the fuzzification while the consequence is the output linguistic variable [13] [14] [15].

Let $F_{\text{longitude}}(m)$ (respectively $F_{\text{latitude}}(m)$) be the fuzzy value corresponding to the longitude (respectively latitude) at month $m$. In this work, we design the fuzzy rules as follows:

- For a model of order 1, the fuzzy rules are constructed as follows:
  IF " $F_{\text{longitude}}(m)$, $F_{\text{latitude}}(m)$ " THEN " $F_{\text{longitude}}(m+1)$, $F_{\text{latitude}}(m+1)$ ".

- For a model of order 2, the fuzzy rules are constructed as follows:
  IF " $F_{\text{longitude}}(m-1)$, $F_{\text{latitude}}(m-1)$, $F_{\text{longitude}}(m)$, $F_{\text{latitude}}(m)$ " THEN " $F_{\text{longitude}}(m+1)$, $F_{\text{latitude}}(m+1)$ ".

- For a model of order 3, the fuzzy rules are constructed as follows:
  IF " $F_{\text{longitude}}(m-2)$ and $F_{\text{latitude}}(m-2)$, $F_{\text{longitude}}(m-1)$ and $F_{\text{latitude}}(m-1)$, $F_{\text{longitude}}(m)$, $F_{\text{latitude}}(m)$ " THEN " $F_{\text{longitude}}(m+1)$ and $F_{\text{latitude}}(m+1)$ ".

The following tables show us examples of fuzzy rules construction according to our methodology adapted with the example partition in Figure 2.a and Figure 2.b.
Table 2: Unclear rules of order model 1 and order model 2

<table>
<thead>
<tr>
<th>Month</th>
<th>Long (East)</th>
<th>F(Long)</th>
<th>Lat (South)</th>
<th>F(Lat)</th>
<th>Order 1 Fuzzy rule</th>
<th>Order 2 Fuzzy rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>97</td>
<td>A3</td>
<td>-33</td>
<td>B2</td>
<td></td>
<td>if F(Long) is A3 and F(Lat) is B2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>then F(Long) is A3 and F(Lat) is B2</td>
<td></td>
</tr>
<tr>
<td>Febr</td>
<td>85</td>
<td>A3</td>
<td>-30</td>
<td>B2</td>
<td></td>
<td>if F(Long) is A3 and F(Lat) is B2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>then F(Long) is A3 and F(Lat) is B2</td>
<td></td>
</tr>
<tr>
<td>Mar</td>
<td>71</td>
<td>A2</td>
<td>-28</td>
<td>B3</td>
<td></td>
<td>if F(Long) is A3 and F(Lat) is B2</td>
</tr>
</tbody>
</table>

Table 3: Unclear rules of order model 3

<table>
<thead>
<tr>
<th>Month</th>
<th>Long (East)</th>
<th>F(Long)</th>
<th>Lat (South)</th>
<th>F(Lat)</th>
<th>Order 3 Fuzzy rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>97</td>
<td>A3</td>
<td>-33</td>
<td>B2</td>
<td>if F(Long) is A3 and F(Lat) is B2</td>
</tr>
<tr>
<td>Febr</td>
<td>85</td>
<td>A3</td>
<td>-30</td>
<td>B2</td>
<td></td>
</tr>
<tr>
<td>Mar</td>
<td>71</td>
<td>A2</td>
<td>-28</td>
<td>B3</td>
<td></td>
</tr>
<tr>
<td>Apr</td>
<td>45</td>
<td>A1</td>
<td>-29.5</td>
<td>B3</td>
<td></td>
</tr>
</tbody>
</table>

2.3.3. The defuzzification

This last step consists in converting the fuzzy linguistic variables resulting from the application of fuzzy rules on the linguistic variables into real values [15]. Among the different defuzzification methods, we used the center of gravity method (Centre or Gravity or COG) [18]. With this method, the real value after the defuzzification operation is equal to:

\[
x_D = \frac{\sum_{i=1}^{n} x_i \cdot \mu_A(x_i)}{\sum_{i=1}^{n} \mu_A(x_i)} \tag{3}
\]

2.4. Evaluation of model [16]

Each partition number pair and model order correspond to a forecast model the Mascarene anticyclone trajectory. Among the possible models, the one that reproduces the observation data most faithfully is the best. Thus, we proceed to
evaluate the difference between the observed and simulated data. To do this, we use the MAPE (Mean Absolute Percentage error) metric. The MAPE metric has the expression:

\[
MAPE = \frac{100}{N} \sum_{t=1}^{N} \left| \frac{A_t - P_t}{A_t} \right|
\]  

(4)

With

- \(A_t\): Observation data at time \(t\);
- \(P_t\): Data forecasts at time \(t\);
- \(N\): total number of observations.

2.5. Choice of model parameters

The simulation consists in varying the partitions number and the models order until a model corresponding to the minimum error is obtained (MAPE). We simulated by varying the number of partitions from 20, 30, 40, 50 and 60 as well as the model order from 1 to 3.

3. The Discourse Universe

The discourse universe \(U\) that we have taken is the real interval between the center minimum and maximum of the anticyclone according to the longitude and the latitude.

Table 4: Discourse Universe \(U\)

<table>
<thead>
<tr>
<th>Center of the anticyclone</th>
<th>Discourse Universe (U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitude</td>
<td>[42.397]</td>
</tr>
<tr>
<td>Latitude</td>
<td>[-35 -28]</td>
</tr>
</tbody>
</table>

3.1. Result of the Choice of the Model Parameters

Figure 3 summarizes the MAPE percentage for each simulation. According to these figures, for each case, the curves have an absolute minimum. Thus, we retain 50 as the partition number and model order 2.
3.2. Result of the Modeling

The Figure 4 shows the best model of the Mascarene anticyclone center trajectory with partition number 50 and order 2. The observation and forecast curves are almost identical.

4. Discussions

Various fields use the fuzzy inference system for modeling [12] [13] [14] [15]. In our study, we use it to model the trajectory of the Mascarene anticyclone center the same input and outputs variables.
The partitions numbers and the model orders for the geographic coordinates of the anticyclone centers are the same. We used the MAPE to validate our model with 10% error margin.

In others works, the number of input and output parameters are different, the MAE (Mean absolute error) and the RMSE (Root-Mean-Square Error) are other means for model validation [17] [18].

5. Conclusion

In this work, we used a fuzzy inference system to model the Mascarene anticyclone trajectory center. The simulations were performed by varying the partitions number to 20, 30, 40, 50 and 60 and the model order from 1 to 3. In each operation, we first defined the discourse universe [42,3 97] for longitude and [-35 -28] and for latitude with 10% safety margin.

The originality of our work is the way the fuzzy rules are constructed, inspired by those used in fuzzy time series theory.

After calculating the error for each simulation, we obtain the best model with a second order model with 50 partitions. In this case, the two observation curves and the model are almost merged.

Conflicts of Interest

The authors declare no conflict of interest.

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