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A Compressive Sensing Scheme Based on Zadoff-Chu Measurement Matrix

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***Abstract*—This paper proposes a compressive sensing framework based on encrypted discrete cosine transform (DCT) sparse and encrypted measurement matrix using partial chaotic Zadoff-Chu matrix transform (ZCMT) for image data. In our proposed scheme, a ZCMT is encrypted by a chaotic sequence, in which the initial value is generated by hashing the transformed image data. In the receiver side, the conventional OMP algorithm is employed to recover the original image data. The PSNR and security performances are evaluated by computer simulation. The simulation results are show that the proposed secure CS scheme has better performances that those of the other measurement matrices schemes.**

Keywords—Compressive sensing, Chaos, Zadoff-Chu matrix transform, measurement matrix, sparse basis matrix

I. INTRODUCTION

With the development of the internet network, there are a lot of images to processed and transmitted. Therefore, a lot of storage space is required in transmission systems. Compressive sensing (CS) has extensively applied to various fields, such as image processing, wireless communications, channel estimation, and wireless sensor network, etc [1-2]. CS theory states that natural signals, which are either sparse or compressible, can be compressed sampled at a rate lower than the Nyquist rate [3]. The resulting compressive data can faithfully recover the original signal. In order to faithfully recovered the original signal, many CS schemes have been proposed. In this work, we mainly focus on image-based compressive sensing. The performance of an image-based CS is depended on the choice of the measurement matrix. in [4], the author proposed an measurement matrix based on chaotic sequence. In [5], a partial chaotic Hadamard measurement matrix was proposed. In reference [6], a performance comparison of measurement matrices was done in term of the recover error, processing time, and recover time, etc. These measurement matrices are random Gaussian matrix, partial Hadamard matrix, Toeplitz matrix, etc. However, the measurement matrix based on Zadoff-Chu matrix (ZCM) has not been considered. Zadoff-Chu matrix ZCM is usually used in OFDM systems, in which ZCM is used to reduce PAPR and improve BER performance of systems [7-8].

ZCM is not caused attention to serve as measurement matrix in image-based CS framework. In this work, we apply ZCM to serve as a measurement matrix in image-based CS framework. The analysis and simulation results show both that the Zadoff-Chu measurement matrix has better performances in term of PSNR merit in image-based CS compared to the other conventional measurement matrices.

II. THE PROPOSED SECURE CS SCHEME

A. Compressive Sensing

Assuming that a discrete ignal $\mathbf{x} \in \mathbf{R}^N$ is *K*-sparse under some sparse basis Ψ, such as DWT or DCT sparse. The sparse representation can be expressed as

$$\mathbf{x} = \Psi \mathbf{s} \tag{1}$$

Where is the transform coefficient vector, which has *K* non-zero elements. Given a measurement matrix $\Phi \in \mathbf{R}^{M \times N}$, the CS theory states that $\mathbf{x}$ can be acquired by the measurement matrix Φ according the following equation (2):

$$\mathbf{y} = \Phi \mathbf{x} \tag{2}$$

Where $M < N$, and $\mathbf{y} \in \mathbf{R}^M$ is call the measurement vector. The original signal $\mathbf{x}$ can be recovered by some reconstructed algorithm based on following l_1 -minimization of the transform coefficient vector $\mathbf{s}$:

$$\min \|\mathbf{s}\|_1 \; s.t. \quad y = \Phi \Psi \mathbf{s} \tag{3}$$

There are many reconstructed algorithms to recovered the original signal $\mathbf{x}$. These algorithms includes linear programming, some convex optimization algorithms, and a family of greedy pursuit algorithms. In this work, the orthogonal matching pursuit (OMP) algorithm [9] is used to evaluate and compare the performance of different measurement matrices.

B. Zadoff-Chu measruement matrix

Zadoff-Chu matrix is generate from ZC sequence. A standard ZC sequence with size of *L* is defined as follows:

$$p(n) = \begin{cases} \exp\left(-j\dfrac{\pi u n(n+1)}{L}\right) & for\ L \quad odd \\ \exp\left(-j\dfrac{\pi u n^2}{L}\right) & for \quad L \quad even \end{cases} \tag{4}$$

where $j=\sqrt{-1}$, u is the root index and it holds that $0<u<L$. In order to generate a ZCMT matrix **P**, the ZC sequence with length of $L=N^2$ is reshaped by $k=m\cdot N+l$,

$$\mathbf{P}=\begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0(N-1)} \\ p_{10} & p_{11} & \cdots & p_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ p_{(N-1)0} & p_{(N-1)1} & \cdots & p_{(N-1)(N-1)} \end{bmatrix} \quad (5)$$

Where m is m and l are the row, column indexes respectively. Therefore, measurement matrix based on ZCM can be expressed as following[7]:

$$\Phi=\sqrt{\frac{N}{M}}\mathbf{DPR} \quad (6)$$

Where $D\subset\mathbb{R}^{M\times N}$ states a subsampling matrix/opertator, $\mathbf{R}\subset\mathbb{R}^{N\times N}$ denotes a permutation matrix, and $\mathbf{P}\subset\mathbb{R}^{N\times N}$ denotes the resulting ZCM matrix according to the equation (5). Based on CS theory, the measurement matrix Φ should be highly incoherent with sparse basis matrix Ψ. The incoherence between the two matrices, can be evaluated according to the definition of the mutual coherence coefficient [8], which can be expressed as

$$\mu(\Phi,\Psi)=\max_{1\le i,j\le N}\left|\langle\Phi_i^T,\Psi_j\rangle\right| \quad (7)$$

Where Φ_i are rows of Φ and Ψ_j are columns of Ψ, respectively.

C. The proposed compressive sensing scheme

Fig .1 shows the proposed image-based compressive sensing scheme based on combing DCT sparse basis and partial Zadoff-Chu measurement matrix. In our proposed scheme, a chaotic sequence generated from Logistic map is employed to achieve row permutation operation of the resulting ZCMT matrix. Chaotic Logistic map is defined as following:

$$y(n+1)=uy(n)(1-y(n)) \quad (8)$$

where $y(n)\in(0,1)$, and $\mu\in(3.75,4]$ is control parameter. $y(0)$ denotes initial value of chaotic map. Initial value x_0 and μ are regarded as the secret keys. Based on equation (8), a chaotic sequence with size of N can be obtained. After that, an integer chaotic sequence can be obtained according to the following processing:

$$z(n)=\text{mod}(\,floor(y(n)+100)*10^{10}),N) \quad (9)$$

Where $\text{mod}(x,N)$ returns the remainder of x divided by *N*, and the function $floor(x)$ returns the value of x to the nearest integer less than or equal to x.

The obtained chaotic sequence $\mathbf{z}=[z(1)\quad z(2)\quad ...\quad z(N)]$ with size of N is used to permute the row indexes of the conventional ZCMT matrix. Assuming that $\mathbf{P}$ is the conventional ZCMT, which can be expressed as following:

$$\mathbf{P}=\begin{bmatrix} p(1) \\ p(2) \\ \vdots \\ p(N) \end{bmatrix} \quad (10)$$

where $\mathbf{P}(i)$ states the *i*-th row vector. After row permutation, the scrambled ZCMT can be expressed as

$$\mathbf{P}^{(1)}=\begin{bmatrix} p(z(1)) \\ p(z(2)) \\ \vdots \\ p(z(N)) \end{bmatrix} \quad (11)$$

The measurement matrix Φ consists of the former M (M<N) rows. It can be expressed as

$$\Phi=\begin{bmatrix} p(z(1)) \\ p(z(2)) \\ \vdots \\ p(z(M)) \end{bmatrix} \quad (12)$$

The signal processing steps of the proposed CS scheme can be summarized as follows:

Step 1: Sparse transformation. Image is usually sparse signal in a DCT sparse basis. So, an original image with a size of $N\times N$ is first transformed into new transformation domain data by a sparse basis Ψ.

Step 2: Sampling. The resulting image data is sampled by the ZCM measurement matrix Φ with $M\times N$. The measurement matrix is constructed according to Eq. (6).

Step 3: Recovering image. In the receiver side, OMP algorithm is applied to reconstructed the image data.

Step 4: Inverse sparse transformation. The obtained image data are recovered to original mage by an inverse sparse basis matrix Ψ^T.

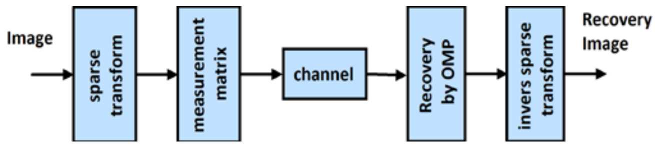

Fig. 1. The propossed compressive sensing scheme based on ZCM Measurement matrix

In order to evaluate the performance of CS based on ZCMT measurement matrix, some conventional measurement matrices, such as partial Hadarmard measurement matrix, partial DCT measurement matrix, and Toeplitz measurement matrix, are used to achieve the performance comparison. For fair compassion, we only use the OMP reconstruction algorithm [10] to recover original image data.

III. SIMULATION RESULTS

In this section, the CS performance is evaluated in terms of peak Signal-to-Noise Ratio (PSNR). PSNR is usually defined as

$$PSNR=10\log_{10}\frac{(2^B-1)^2}{MSE} \quad (13)$$

where $MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left\|I(i,j)-K(i,j)\right\|^2$ and B states the bit numbers per pixel. The higher the PSNR the better the quality of the recovered image. In the following simulation experiments, the DCT sparse basis matrix is used. We first compared the performance of our proposed ZCMT measurement matrix with those of the other conventional measurement matrices. In the following experiments, the compressive ratio is defined as

$$\text{compressive ratio}=M/N \tag{14}$$

where M and N are the numbers of row and column of measurement matrix, respectively.

Fig. 2 shows the PSNR performance of the recovered images under different measurement matrices and different compressive ratio values. Cameraman image with size of 256×256 is served as test image. in addition, for the Logistic map, the initial value y(0) and control parameter u are fixed at 0.33 and 4. From the Fig. 2, it can be seen that the performance of the ZCMT measurement matrix has almost the same as that of the DFT measurement matrix, and has better than those of other measurement matrices, such as DCT, Hadamard, DHT, Toeplitz, and Bernoulli measurement matrices. It also can be seen that the PNSR become bigger with the increasing of the compressive ratio value. The performances of the DCT, DHT, and Hadmard measurement matrices is almost the same. In addition, the performance of the Toeplitz measurement matrix has also the similar performance of the Bernoulli measurement matrix. At the compressive ratio of 0.6, the ZCMT measurement matrix can obtain 2 dB, and 3 dB PSNR gains when compared to that of Hadamard measurement matrix and that of Toeplitz measurement matrix, respectively.

Fig. 3 shows the reconstructed images with different measurement matrices under compressive ratio of 0.5. Fig. 3 (a) is the original Cameraman image. Fig. (b) is the recovered image with ZCMT measurement matrix. Fig. 3 (c), (d), (e), (f), (g), and (h) are the reconstructed images with Hadamard, DCT, DHT, DFT, Toeplitz, and Bernoulli measurement matrix, respectively. From the Fig. 3, we can see that the quality of reconstructed image based on ZCMT and DFT is the best.

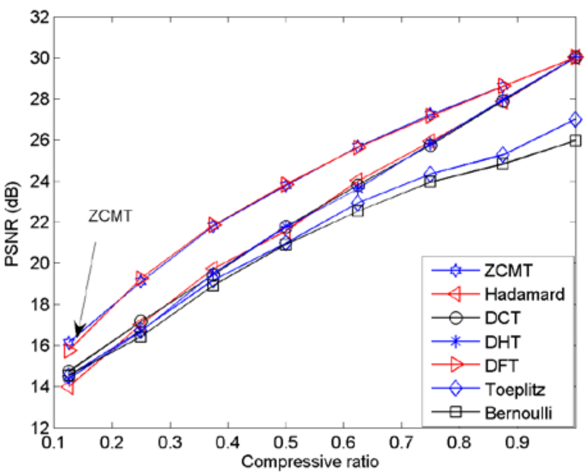

Fig.2. PSNR performance comparison for different measurement matrices

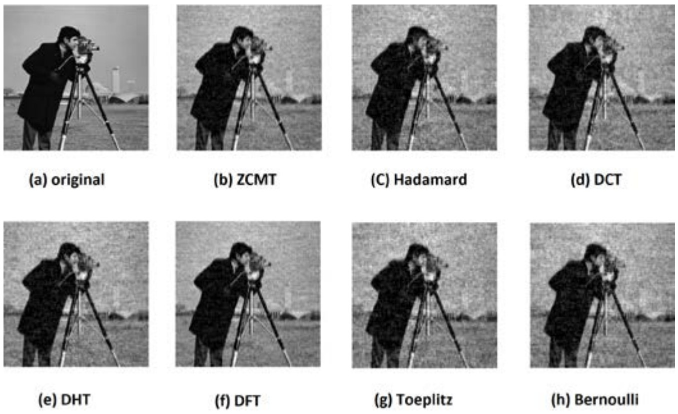

Fig.3. Recovered images by different measurement matrices under compressive ratio of 0.5.

In the following, we evaluated the PSNR performance of the ZCMT measurement matrix using different images. The four images include Cameraman, Lena, Pirate, and Woman-blonde images. The size of every image is 512×512.

Table. 1 shows the experiment results using ZCMT measurement matrix at compressive ratio of 5/8. And Fig. 4 shows the recovered images. From Table. 1 and Fig. 4, we can see that the recovered quality of Lena image is the best. The MSE of Lena image is least. The run time and MSE are also shown in Table. 1. However, the different of the recovered images is very small.

Table 1. PSNR Performance comparison of different image under the given compressive ratio (5/8)

Image	PSNR(dB)	Time (s)	MSE
Cameraman	31.7800	91.8278	1.1314e+07
Lena	32.5639	134.4976	9.4456e+06
Pirate	29.0846	146.9877	2.1045e+07
Woman_blonde	28.8441	150.3384	2.2244e+07

Fig.4. PSNR performance comparison for different measurement matrices

IV. CONCLUTIONS

In this paper, we proposed a image-based compressive sensing scheme based in ZCMT measurement matrix. The performance of CS was evaluated and compared with other conventional measurement matrices. The simulation results show that the performance of the ZCMT is better than those of other measurement matrices with the exception of the DFT matrix.

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