A pricing strategy for sharing autonomous vehicle system

Ming Li*
School of Transportation Science and Engineering
Beihang University
Beijing, China
*Corresponding author

Xiang Huo
School of Transportation Science and Engineering
Beihang University
Beijing, China

Abstract—**Autonomous vehicles (AVs) are regarded as a promising transportation tool of future that provides increased traffic mobility, enhanced customer satisfaction and reduced infrastructure costs. Since AVs follow completely the system directions, it is possible to further optimize the itineraries for an even higher efficiency than transportation system with traditional vehicles. In managing the system, dispatching the SAVs and controlling the total car number are the primary concerns. A pricing strategy should also be considered for promoting passengers' adoption of the shared autonomous transportation. To this end, a set-covering based model is proposed to link charging fares and vehicle-and-passenger assignment. For the NP-hard problem in searching for feasible routes, a column generation based algorithm is introduced. At last, the effectiveness of the heuristics algorithm is tested by a numerical example, and the sensitivity of parameters are also analyzed.**

Keywords—Shared autonomous vehicle, ride-sharing, pricing.

I. INTRODUCTION

In recent years, the transportation system of autonomous vehicles (AVs) has been paid much attention because it has the potential of improving traffic efficiency and mobility. For AVs, Levin stated that one shared autonomous vehicle (SAV) has the ability of replacing between 3 and 11 private vehicles[1]. Furthermore, AVs could be operated without drivers, therefore labor cost of SAVs caused by relocating vehicles would be significant reduced. With these momentums, it is not difficult to see that SAVs will become one of the promising business models. In addition to SAVs, ride-sharing is another critical way to increase vehicle occupation rates and save on-road vehicles[2]. Under the background of multi-sharing schemes, i.e., SAVs and ride-sharing, the AV-based transportation system might make great contribution on releasing traffic congestion. Thus, how to design an effective AV-based serving system will become an important issue to be addressed.

With the development of information and communication technologies, it will become realization in the near future that a central control center conducts AV paths by maximizing the total revenue to meet passenger travel demands. Till now, this kind of problem is usually taken as a vehicle routing problem [3] or dial-a-ride problem[4]. And most existing researches focus on minimizing the total travel distance to match vehicles and passengers, e.g.,[5,6]. However, fare prices for passengers are seldom discussed. To narrow this gap, current paper intends to explore the impact of prices on

vehicle routing plans and then provide a holistic method to simultaneously optimize serving routes and prices. Since ride-sharing is projected in the AV-based transportation system, serving prices should not only be related with passenger travel distances. A cost-sharing mechanism needs to be added to determine the optimal price. Furthermore, the vehicle routing optimization for multiple passengers has been considered as a NP-hard problem. Therefore a cost-sharing strategy and route optimization method will be simultaneously investigated for the SAV system. The major points of this paper intend to answer the following questions as summarized as follows:

How to match passengers who share a ride and determine competitive fare prices?

How to balance fare prices and the system total cost?

How to construct an algorithm to dispatch AVs with a minimize number of vehicles for improving traffic jam?

The remainder of this paper is organized as follows. A detailed description of the problem and the optimization model for the problem are presented in Section II and III. Then the optimization algorithm is developed in Section IV. Finally, some numerical results and concluding remarks are summarized in Section V and VI.

II. PROBLEM STATEMENT

In this paper, we try to dispatch SAVs while maximizing the system benefit and minimizing passengers payoff cost. For the system, high fare prices are profitable, but they are not unwilling for passengers. Furthermore a discount should be considered for passengers sharing a ride. Thus the main challenge is to provide an incentive-compatible price to promote the system providing the ride-sharing service and passengers accepting it. Fare prices for non-sharing passengers are usually consisted by flag-down fare and distance fare, while there is not a specific scheme for carpool passengers. Thus a cost-sharing pricing strategy should be researched for the system and passengers to make decisions. To make fare prices reasonable, this paper will proposed a holistic pricing method by simultaneously considering waiting time, travel delay, the number of carpool passengers and vehicle cost. With the pricing method, the system benefit and passenger cost could be obtained, where the system benefit denotes the total income minus vehicle cost. For simulating the system reliably, depreciation, maintenance and fuel cost are considered when estimating vehicle cost.

For vehicle cost, depreciation and maintenance cost could be regarded as an identical value for each vehicle, while fuel cost varies with vehicles' travel distance. In this way, it implies that SAV serving routes will have an impact on fuel cost. Since a passenger's feasible travel trajectories might be crossed with many passengers, numerous candidate routing plans will be generated. Thus it is difficult for the system to find serving routes with a maximizing benefit. With the purpose of simplifying routing sets, this paper will explore how to produce feasible serving routes and update them in an effective way.

Based on these considerations, we first introduce the incentive-compatible priority pricing mechanism by modeling an M/M/1 queuing system. Then a set-covering model is built for guiding AVs' serving routes, which takes visiting nodes of each vehicle as a variable. To compute the proposed model, the column generation is applied to explore promising serving routes by solving a sub-problem in the dual-space. Finally, a SAV route planning algorithm will be developed through integrating the pricing method and column generation.

III. MATHEMATICAL MODELLING

Let U be the set of passengers. For the control center, we assume that each passenger u will submit its specific origin-destination locations (i.e., $O(u)$ and $D(u)$) to the system. In addition, each passenger is expected to be served at a fixed time, i.e., pick-up time windows [ϕ_u^e, ϕ_u^l] and drop-off time windows [φ_u^e, φ_u^l]. In general, the vehicle dispatching problem is defined on a complete graph $G = (V, A)$, as shown in Fig. 1, where the travel time denotes the edge weight, e.g., $\Gamma_{O(1),D(2)}$. For simplicity, travel time between any two nodes is considered constant and the influence of traffic congestion is omitted. The objective is to achieve a win-win situation (i.e., maximize the system profit and minimize passenger cost) by providing the ride-sharing service. The decisions variables are

p_u fare price for passenger u

x_m a binary variable, 1 if the serving route m is adopted and 0 otherwise

t_u^o the prick-up time for passenger u

t_u^d the drop-off time for passenger u

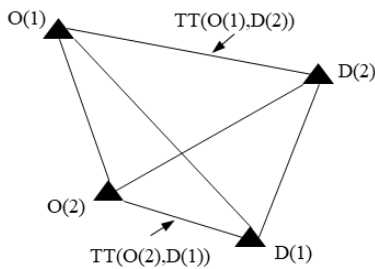

Figure 1 The sketch network

Before modeling the optimization problem, some requirements are listed as follows

1) All passengers should be served;

2) Each passenger is only served by one AV;

3) Passenger departure-arrival time windows should be strictly satisfied;

4) Passengers are only picked up and dropped off at their submitting locations;

For each passenger, the fare price of non-carpooling is presented as Eq. (1), while the price of ride-sharing is influenced by delayed time and the number of passengers. It is obvious that the price of ride-sharing is lower than that of non-carpooling. Thus Eq.(1) gives the upper bound of an actual price charged by the system.

$$p_u^+ = a_0 + c_0 \cdot \max\{0, d_u - d_0\} \tag{1}$$

where a_0 denotes the flag-down price; c_0 represents distance fare; d_0 is the basic distance; d_n is the actual travel distance.

For a SAV, c_v is the fixed cost (depreciation and maintenance cost) per day and c_f is the fuel cost per mile. In this way, the AV-based transportation system with the ride-sharing service is formulated as follows:

$$\max \sum_{m \in \Omega} \sum_{u \in \pi^m} x_m \cdot p_u - \sum_{m \in \Omega} x_m \cdot c_f \cdot r_m - \sum_{m \in \Omega} x_m \cdot c_v \tag{2}$$

s.t.

$$\sum_{m \in \Omega} x_m \cdot y_m^u = 1 \quad \forall u \in Q \tag{3}$$

$$\left| t_i^o - t_j^o \right| \geq \Gamma_{O(i),O(j)} + (2 - y_m^i - y_m^j) \cdot M \quad \forall i, j \in \pi^m, m \in \Omega \tag{4}$$

$$\left| t_i^o - t_j^d \right| \geq \Gamma_{O(i),D(j)} + (2 - y_m^i - y_m^j) \cdot M \quad \forall i, j \in \pi^m, m \in \Omega \tag{5}$$

$$\left| t_i^d - t_j^d \right| \geq \Gamma_{D(i),D(j)} + (2 - y_m^i - y_m^j) \cdot M \quad \forall i, j \in \pi^m, m \in \Omega \tag{6}$$

$$\phi_u^e \leq t_u^o \leq \phi_u^l \quad \forall u \in Q \tag{7}$$

$$\varphi_u^e \leq t_u^d \leq \varphi_u^l \quad \forall u \in Q \tag{8}$$

$$\xi_u^- \leq p_u \leq \xi_u^+ \quad \forall u \in Q \tag{9}$$

$$x_m \in \{0,1\} \tag{10}$$

The objective function Eq.(2) maximizes the total profit, where Ω is the set of all feasible serving routes and π^m is the set of passengers in the m-th feasible serving route. Constraints (3) ensure that each passenger only served by a vehicle, which also implies that transfer is not allowed. Constraints (4-6) denote that serving time of passengers carried by the same vehicle should be restricted with time difference in the temporal-and-spatial space. Constraints (7-8) respectively represent the upper and lower bound of pick-up and drop-off time. Constraints (9) give the range of fare prices, where the maximum price could be estimated by Eq. (1) and the minimum value might be a variable or a fixed parameter. For simplify, we assume the minimum price is an identity number for each passenger, which is determined by the system cost.

Compared with the node-based vehicle routing optimization model, the set-covering optimization model is simpler. In this model, we assume that we know the feasible

routing set Ω, which is difficult to be obtained in reality especially for numerous passengers. Thus the proposed model is not possible to be directly solved by an optimization solver. An efficient algorithm should be introduced to find optimal solutions, which will be illustrated in the following section.

IV. SOLUTION METHOD

Since the number of passengers served by a vehicle may not be the same, the length of visiting nodes will vary with it. This brings obstacle to present serving routes in a unified form. To simplify this expression, we deliver vehicle tasks by only revealing passengers' ID number instead of detailed serving routes, as shown in Fig. 2. Each column represents a feasible plan for a vehicle, e.g., the m-th column implies that passenger 1, 3 and 5 could be served by a vehicle without time constraints. Meanwhile the total benefit for each column is derived with Eq. (2). Therefore, the critical issue is how to determine the number of feasible columns K and find optimal columns to maximize the total benefit with constraints (3). To solve these problems, we first introduce the incentive-compatible priority pricing mechanism for evaluating ride-sharing passengers' fare prices, then the column generation algorithm is adopted to generate new columns for enlarging the number of columns K. The proposed algorithm process is concluded as follows.

Figure 2 Column description for the vehicle dispatching problem

A. The incentive-compatible priority pricing

Mendelson and Whang [7] proposed a pricing mechanism for multiple user classes by modeling an M/M/1 queue system, which could generate an incentive-compatible price to induce users' behaviors. In this system, each user u was characterized by delay cost c_u, expected service time w_u, arrival rate λ_u and value function $h_u(\lambda_u)$. If the relationship between the value function and price satisfies Eq. (11), the system is considered at the equilibrium.

$$h_u(\lambda_u) = p_u + c_u \cdot G_u(\underline{\lambda}) \tag{11}$$

where p_u denotes the charged price and $G_u(\underline{\lambda})$ is the expected time spent in the system, where $\underline{\lambda}$ presents the arrival rate vector.

Since this price mechanism aims at optimizing the whole benefit of the system, the objective function is formulated as Eq. (12),

$$\max \sum_{u \in U} \int_0^{\lambda_u} h(\lambda_u) - \sum_{u \in U} c_u \lambda_u G_u(\underline{\lambda}) \tag{12}$$

By taking the derivative of Eq. (12) with respect to λ_u, the optimal price $\tilde{p}_u$ for each user u could be represented as Eq. (13).

$$\tilde{p}_u = \sum_{i=1}^{U} c_u \tilde{\lambda}_i \frac{\partial G_i(\underline{\lambda})}{\lambda_u} \quad \forall u \in U \tag{13}$$

More detailed information could be found in [7].

B. The column generation-based algorithm

The advantage of the column generation algorithm is that it does not need to enumerate all possible columns, e.g., K in Fig. 2 may not be a very large number. It usually tackles the optimization model by solving two problems, i.e., the restricted master problem and the sub-problem. Hence, the process of the column generation-based algorithm could be concluded as initial columns generation, finding columns, which is listed as follows.

Step 1 Given an initial set of feasible columns, i.e., the matrix A;

Step 2 Calculate revenue for each column by Eqs. (2) and (13).

Step 3 Solve the restricted master problem and calculate the dual multiplier τ.

Step 3 Adopt dynamic programming to obtain optimal a_i of the sub-problem, which is considered as a knapsack problem.

Step 4 Add the new column a_i into the original matrix A

Step 5 If the objective value of the sub-problem is a non-negative number, terminate and output solutions; otherwise return to step 2.

V. CASE STUDY

The numerical test is conducted on a simple grid network of 5×5. Travel time of each link between adjacent nodes is assumed to be 60 seconds. Traveler OD pairs and time windows are generated randomly. For price parameters, flag-down price is set to 11$, distance fare is 1.5$/mi and the basic distance is 3mi. For vehicles, fixed cost is set to 50$ and fuel cost is 0.2$/mi.

For simplicity, the optimization model is solved with a set of 20 travelers. The required number of vehicles is 6 without the consideration of price. Due to vehicles fixed cost, the optimal number of vehicles for the proposed model is also 6 for saving the operation cost, as listed in Table I. With the incentive price, the column generation-based algorithm tends to assign travelers with similar demands into a car. Furthermore, the occupation rate of vehicles could be improved.

Table I Vehicle-and-passenger assignment

Vehicle ID	Passengers
1	4→3→2→1→-4→-3→-2→-1
2	5→18→17→16→-5→-18→-17→-16
3	6→7→8→9→-9→-8→-6→-7
4	10→13→12→11→-10→-12→-11→-13
5	14→15→-14→-15
6	19→20→-19→-20

By adjusting the fixed cost, it seems that the proposed model has a risk in increasing the number of vehicles to raise the total benefit, if it is close to 0. In addition, results are highly affected by demand distribution. The incentive pricing strategy will make a little influence on both the system and travelers, when only a few of passengers have the possibility to travel with others.

VI. CONCLUSION

In this paper, a sharing transportation system is built to dispatch vehicles and determine charging fares. To model this problem, a set-covering method is adopted to represent the relationship between vehicles and passengers. Then an incentive pricing strategy is introduced to evaluate discount values for passengers. To solve this model, a column generation based algorithm is applied to find optimal solutions. A numerical test is conducted to test the validity and efficiency of the heuristic algorithm. Results show that vehicles fixed cost and flag-down price will make an effect on the number of required vehicles.

Through the whole study, we only consider the incentive pricing method for passengers in a single vehicle, i.e., a single queue. In the future research, it might be extended to consider charging fares and the number of vehicles among multiple vehicles.

REFERENCES

[1] M. W. Levin, "Congestion-aware system optimal route choice for shared autonomous vehicles," *Transportation Research Part C*, vol. 82, pp. 229-247, Septempter 2017.

[2] J. Alonso-Moraa, S. Samaranayake, A. Wallara, E. Frazzolic, and D. Rus, "On-demand high-capacity ride-sharing via dynamic trip-vehicle assignment," *Proc Natl Acad Sci*, vol. 114, pp. 462-467, January, 2017.

[3] M. Lokhandwalaa, H. Cai, "Dynamic ride sharing using traditional taxis and shared autonomous taxis: A case study of NYC," vol. 97, pp. 45-60, October 2018.

[4] J.-F. Cordeau, "A Branch-and-Cut Algorithm for the Dial-a-Ride Problem," *Operations Research*, vol. 54, no. 3, pp. 573-586, May 2006.

[5] H. Hosni, J. Naoum-Sawaya, H. Artail, "The shared-taxi problem: Formulation and solution methods," *Transportation Research Part B*, vol. 70, pp. 303-318, Septempter 2014.

[6] M. Liu, Z. Luo, A. Lim, "A branch-and-cut algorithm for a realistic dial-a-ride problem," *Transportation Research Part B*, vol. 81, pp. 267-288, November 2015.

[7] H. Mendelson, S. Whang, "Optimal incentive-compatible priority pricing for the M/M/1 queue," Operations Research vol. 38, no.5, pp. 870–883, October 1990.