Research and Improvement of the Formula of Correlation Coefficient in Image Processing

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Abstract: The research of sub-pixel algorithm has always been a hot topic in the field of image processing. How to improve the accuracy of sub-pixel algorithm and reduce its error is difficult and difficult. This paper uses the digital speckle correlation method and the formula of the correlation coefficient of different powers to calculate the sub-pixel displacement of the image through MATLAB. The experimental results show that the higher-order formula is more stable and accurate than the lower-order formula. With the increase of the power, the error of the calculation result is smaller, and the error calculated by formula $C^{15}{}_{ZNCC}$ is reduced by 21.66% than the error calculated by formula C_{ZNCC}. Studies have shown that by increasing the number of powers of the correlation coefficient calculation formula can reduce the error of the sub-pixel algorithm.

1. Introduction

In the statistics, there is a certain correlation between independent variables and dependent variables, and the parameters used to describe this correction are called correlation coefficients[1][2]. Prior to the appearance of correlation coefficients, the correlation tables and correlation graphs may reflect the relationship between two variables and their associated directions, but it is not possible to indicate exactly the degree of correlation between the two variables. The three most popular are: Pearson's coefficient, Spearman's rho coefficient, and Kendall's tau coefficient[2]. The most widely used correlation coefficient is Pearson correlation coefficient. Pearson's coefficient of correlation was discovered by Bravais in 1846, but Karl Pearson was the first to describe, in 1896, the standard method of its calculation and show it to be the best one possible[3].The correlation coefficient is an important index to measure the correlation between two images in the field of image processing. So, Its development and improvement are of great significance to the image processing field. After several decades of development, different scholars have studied the correlation coefficient. they are divided into two categories: product correlation and subtraction correlation, according to the different calculation methods[4]. In this paper, we classify and study the correlation coefficients. We not only give the basic forms of the correlation coefficients, but also verify and compare them. In addition, a new improved method is given.

2. Correlation coefficient formula and its improvement

In the field of image processing, through the research, Domestic and foreign scholars have made a comprehensive comparison and analysis of correlation coefficients, which based on cross-correlation and distance. They summarized the multiple correlation coefficient function can be roughly divided into two categories: product related functions and subtraction correlation function. Given that the reference image and the target image are f and g respectively, the specific functions of the correlation coefficients of the two images are as follows[5][6][7]:

2.1 Product correlation function

Cross correlation function (CC):

$$C_{CC} = \sum f(x_i, y_i) \cdot g(x_i^*, y_i^*) \tag{1}$$

Normalized cross correlation function (NCC):

$$C_{NCC} = \frac{\sum f(x_i, y_i) \cdot g(x_i^*, y_i^*)}{\sqrt{\sum f^2(x_i, y_i)} \cdot \sqrt{\sum g^2(x_i^*, y_i^*)}} \tag{2}$$

Zero-mean normalized cross correlation function (ZNCC):

$$C_{ZNCC} = \frac{\sum [f(x_i, y_i) - \overline{f}] \cdot [g(x_i^*, y_i^* - \overline{g})]}{\sqrt{\sum [f(x_i, y_i) - \overline{f}]^2} \cdot \sqrt{\sum [g(x_i^*, y_i^*) - \overline{g}]^2}} \tag{3}$$

Product correlation function is also called cross correlation criterion function, and zero-mean normalized cross correlation function is the most commonly used product correlation function. Its value is between -1 and 1[8]. In image processing, the closer the value approaches to 1, the more relevant the two images are. On the contrary, the more unrelated the two images are. When C_{ZNCC} equals 1, the two images are completely correlated. When C_{ZNCC} is equal to -1, the two images are completely irrelevant.

2.2 Subtraction correlation function

Sum of squared difference function (SSD):

$$C_{SSD} = \sum [f(x_i, y_i) - g(x_i^*, y_i^*)]^2 \tag{4}$$

Normalized sum of squared difference function (NSSD):

$$C_{NSSD} = \sum \left[\frac{f(x_i, y_i)}{\sqrt{\sum f^2(x_i, y_i)}} - \frac{g(x_i^*, y_i^*)}{\sqrt{\sum g^2(x_i^*, y_i^*)}} \right]^2 \tag{5}$$

Zero-mean normalized sum of squared difference function (ZNSSD):

$$C_{ZNSSD} = \sum \left\{ \frac{f(x_i, y_i) - \overline{f}}{\sqrt{\sum [f(x_i, y_i) - \overline{f}]^2}} - \frac{g(x_i^*, y_i^*) - \overline{g}}{\sqrt{\sum [g(x_i^*, y_i^*) - \overline{g}]^2}} \right\}^2 \tag{6}$$

Subtraction correlation function is also called square sum criterion function, and zero-mean normalized sum of squared different function is the most commonly used subtraction correlation function. In image processing, unlike the product correlation function, the closer its value approaches to 0, the more correlated the two images are, otherwise the more unrelated the two images are. When C_{ZNSSD} equals to 0, the two images are completely related.

In the above expression, i and j are in the range [1, m], and the expression of $\overline{f}$ and $\overline{g}$ are as follows:

$$\overline{f} = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} f(x_i, y_i) \tag{7}$$

$$\overline{g} = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} g(x_i^*, y_i^*) \tag{8}$$

Where $f(x_i, y_i)$ and $g(x_i^*, y_i^*)$ denote the gray values of the midpoints of the reference image and the target image, respectively; $\overline{f}$ and $\overline{g}$ are the mean value of gray for the reference image sub area and the target image sub area respectively; $\sum f(x_i, y_i)$ and $\sum g(x_i^*, y_i^*)$ are the cross correlation function of the reference image and the target image before and after the deformation; $\sum f^2(x, y)$ and $\sum g^2(x_i^*, y_i^*)$ are the auto-correlation functions of the reference image and the target image, respectively; $\sum\left[f(x_i, y_i) - \overline{f}\right]$ and $\sum\left[g(x_i^*, y_i^*) - \overline{g}\right]$ are the covariance cross correlation functions of the reference image and the target image before and after the deformation; $\sum\left[f(x_i, y_i) - \overline{f}\right]^2$ and $\sum\left[g(x_i^*, y_i^*) - \overline{g}\right]^2$ are the covariance auto-correlation functions of the reference image and the target image, respectively[9][10][11].

Research shows that there is a relationship between zero-mean normalized cross correlation function and zero-mean normalized sum of squared difference function. We can obtain

$$f' = f(x_i, y_i) - \overline{f} \qquad (9)$$

$$g' = g(x_i^*, y_i^*) - \overline{g} \qquad (10)$$

According to (3), (6), (9) and (10), it is easy to prove that the ZNSSD coefficient is directly related to the ZNCC coefficient as follows.

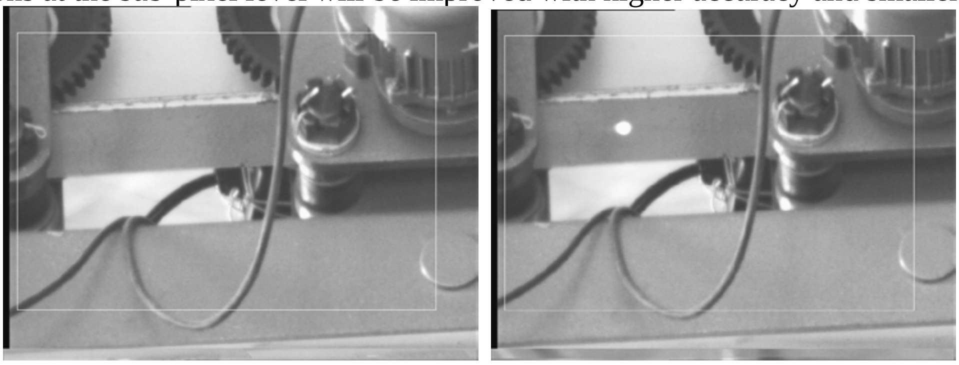
$$\begin{aligned} C_{ZNSSD} &= \sum\left(\frac{f'}{\sqrt{\sum f'^2}} - \frac{g'}{\sqrt{\sum g'^2}}\right)^2 \\ &= \sum\left(\frac{f'^2}{\sum f'^2} - 2\frac{f' \cdot g'}{\sqrt{\sum f'^2}\sqrt{\sum g'^2}} + \frac{g'^2}{\sum g'^2}\right) \\ &= 2 - 2\frac{f' \cdot g'}{\sqrt{\sum f'^2}\sqrt{\sum g'^2}} \\ &= 2(1 - C_{ZNCC}) \end{aligned} \qquad (11)$$

Equation (11) evidently indicates that the ZNSSD and ZNCC criteria are equivalent. In practice, because the computation of ZNSSD coefficient is relatively easier than that of the ZNCC coefficient, ZNSSD criterion is usually employed[12].

Studies have shown that the cross-correlation function criterion and the interpolation sum-square function criterion are sensitive to the influence conditions of light offset, light intensity compensation, light intensity change, and linear amplification; normalized cross correlation function criterion and normalized sum of squared difference function the criteria are sensitive to light intensity compensation and light offset, and are not sensitive to changes in light intensity and linear amplification; and zero-mean normalized cross correlation functions and zero-mean normalized sum of squared difference function are used to compensate light offset and light intensity. Light intensity change and linear amplification are all insensitive, and its anti-noise interference ability is also good. Therefore, improving the ZNCC criterion and the ZNSSD criterion can inherit these advantages.

3. Optimization and Analysis of Formula of Correlation Coefficient

According to the above formula, this paper proposes an improved method based on zero-mean normalized cross correlation function and zero-mean normalized sum of squared difference function. In image processing, we found that the higher the number of correlation coefficient formulas, the sharper the maximum peak, the more obvious the peak value, and the local peak value can be filtered out for a certain number of times, neglecting. In the speckle correlation method, if the correlation coefficient formula is used, the disadvantage of the non-uniform white speckle peak can be compensated, which is similar to the single peak effect of laser speckle. Although there is no influence on the search of the whole pixel, the position of the maximum peak is not it will change, the calculations at the sub-pixel level will be improved with higher accuracy and smaller error.



(a) reference image (b) target image

Fig.1 Image before and after displacement

Where, two digital images of a represent the image processing objects shown in Fig.1. The fig.1(a) shows the reference image, and the fig.1(b) shows the deformed image that is shifted by 19 pixels in the x and 2 pixels y directions by MATLAB in the reference image.

Feel free to take a 41×41 window in Fig.1(a). In Fig.1(b), use the above formulas and, and these formulas to perform matching at the whole pixel level. The peak graphs of different product correlation coefficient diagram are shown in Fig.2.

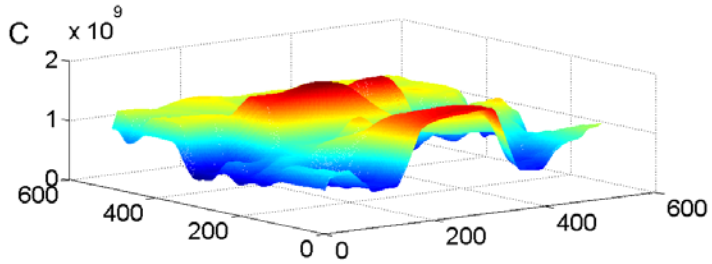
Fig.2 shows the 3-D rendering of the correlation coefficient values for different product correlation formula within a step length of 500. Among them, Fig. 2(a) is the peak value of global correlation coefficient calculated by C_{CC}. Fig. 2(b) is the peak value of global correlation coefficient calculated by C_{NCC}. Fig.2(c) is the global correlation coefficient calculated by C_{ZNCC}.

Fig.3 shows the 3-D rendering of the correlation coefficient values for different product correlation formula within a step length of 500. Among them, Fig.3 (a) is the peak value of the global correlation coefficient calculated by (4). Fig.3 (b) is the peak value of the global correlation coefficient calculated by (5). Fig.3(c) is the global correlation coefficient of (6) calculated.

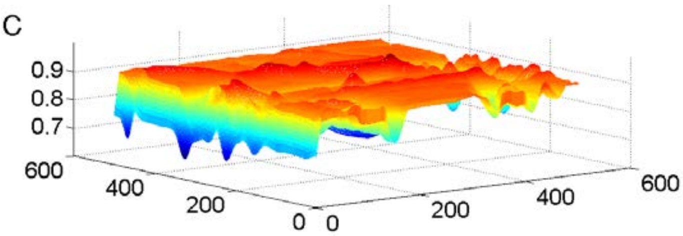
It can be seen from Fig.2 that the zero-mean normalized cross correlation function is the best. Compared with other product correlation function formulas, its main peak is more obvious and sharp, and the difference and distance between the sub-peak and the main peak is larger. From the Fig.2, we can see that in the two types of subtraction coefficient and product correlation coefficient, the function of zero-mean normalized is the best. Compared with other formulas, the main peak is more obvious and sharp, and greater drop and distance between main and secondary peaks. In addition, the three peak graphs of the above different subtraction correlation coefficient diagram are shown in Fig.3. Unlike product correlation function, the greater the value of subtraction correlation function, the more irrelevant it is, and the greater the 0, the more relevant.

Next, we compared the zero-mean normalized correlation coefficients, $C^n{}_{ZNCC}$ and $C^n{}_{ZNSSD}$ with n=2, 5, 10, 15, next. A window is randomly selected in Fig.1 (a), and the window is matched with Zero-mean normalized cross correlation coefficient in Fig.1 (b). The relationship between the matched window and the window in the 100-step-long domain is matched. The correlation coefficient diagram is shown in Fig. 4.

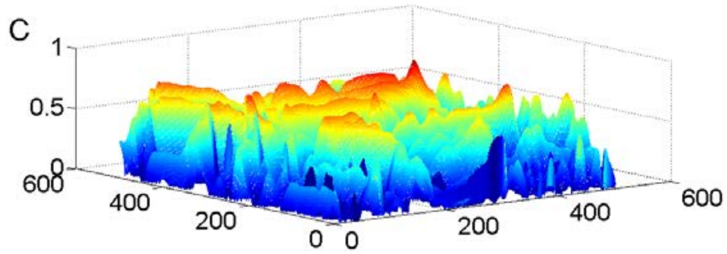
The four graphs in Fig.4 represent the correlation coefficient calculated using the formula $C^n{}_{ZNCC}$ (n=2, 5, 10, 15) and the coordinate points in the two-dimensional direction (x, y direction). From the Fig.4, we can see that the higher the value of n, the more prominent the main peak and the smaller and less the second peak.



(a)

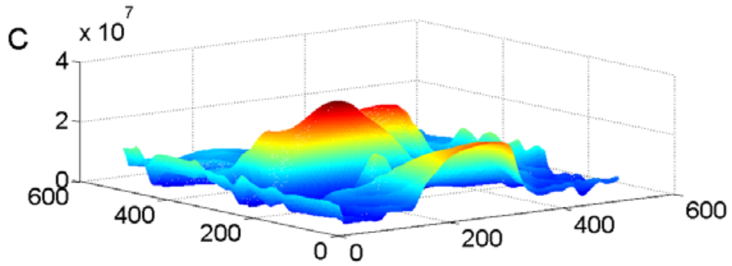


(b)

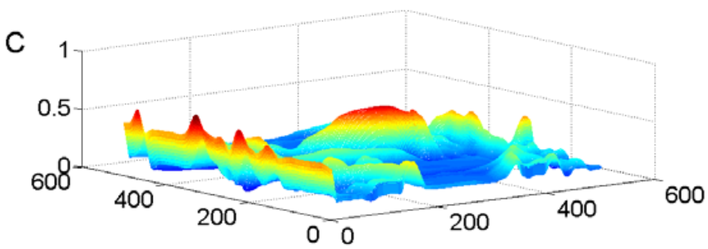


(c)

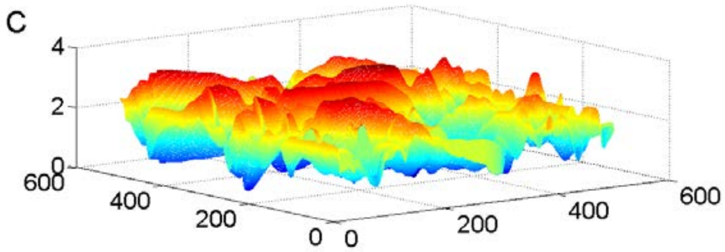
Fig2 Peak graphs of different product correlation function



(a)



(b)



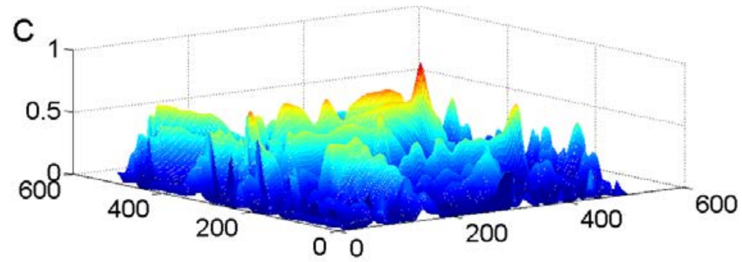
(c)

Fig.3 Peak graphs of different subtraction correlation function

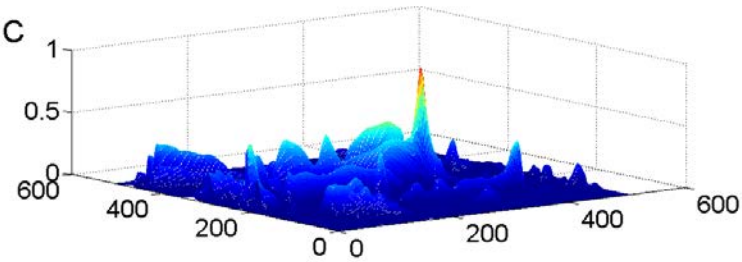
Calculate the correlation coefficient C with different n, and take the correlation coefficient values of 100 points adjacent to the maximum correlation coefficient C on the y axis to compare the trend of the correlation coefficient C values, as shown in Fig. 5.

Fig.5 respectively show the correlation coefficient values of the formulas $C^{n}{}_{ZNCC}$ (n=1, 2, 5, 10, 15) and the coordinate points of the one-dimensional direction (y-direction).

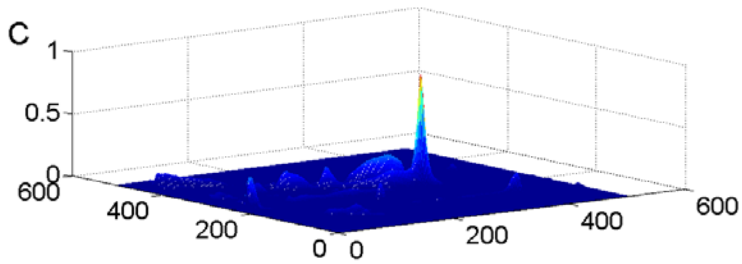
Compared to the different color lines, it is easy to see that the main peak and the secondary peak change in the two-dimensional maps. The main peak becomes more and more obvious, but it is more obvious that the value of secondary peak decreases with the increase of n and even approaches 0. As the n gets bigger, the maximum correlation coefficient C gets smaller and smaller, but it does not change much compared with the decrease of sub-peak.



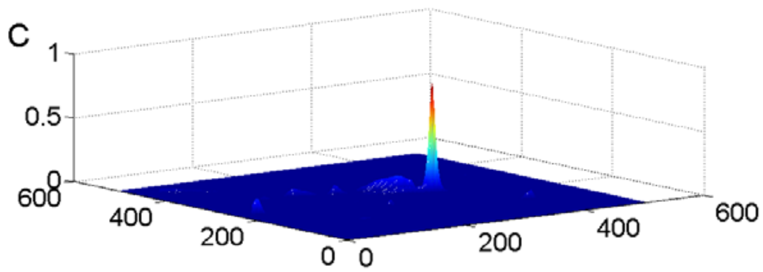
(a)



(b)



(c)



(d)

Fig. 4 peak graphs of ZNCC with different n in two directions

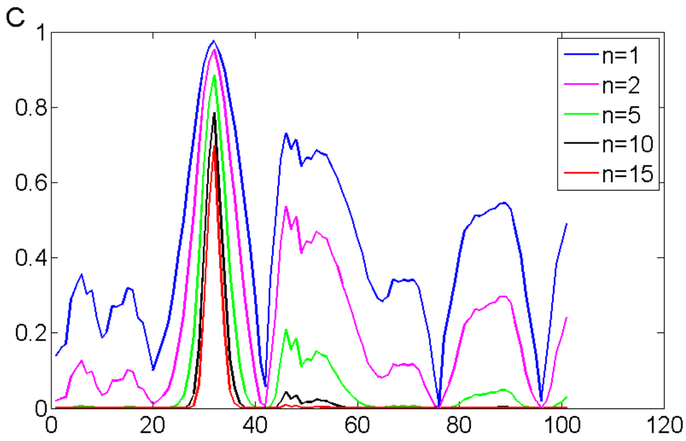


Fig. 5 peak graphs of ZNCC with different n in y-direction

It can be seen comprehensively, from Fig.4 and Fig.5, the value of optimized zero-mean normalized correlation coefficient calculation C^n_{ZNCC} decreases with the increase of n value, but the shape of each peak is more and more sharp, the main peak becomes more and more prominent, and the second peak fewer and fewer, and most of them are filtered out, but the main peaks are getting smaller and smaller, and tend to be closer to 0. Therefore, the value of n also has a certain range. Exceeding this range, not only will the secondary summit be filtered out, but the main peak will also be filtered out, and subsequent calculations will not be possible; and if the number is too high, the amount of calculation will increase and the calculation time will increase.

4. Sub-pixel displacement experiment

After the whole-pixel search, we use C_{ZNCC} and C^{10}_{ZNCC} to calculate the sub-pixel values by quadric surface fitting. Here, we use the surface fitting method that needs to calculate the correlation coefficient to calculate the sub-pixel displacement of the two images.

The formula used by the surface fitting method:

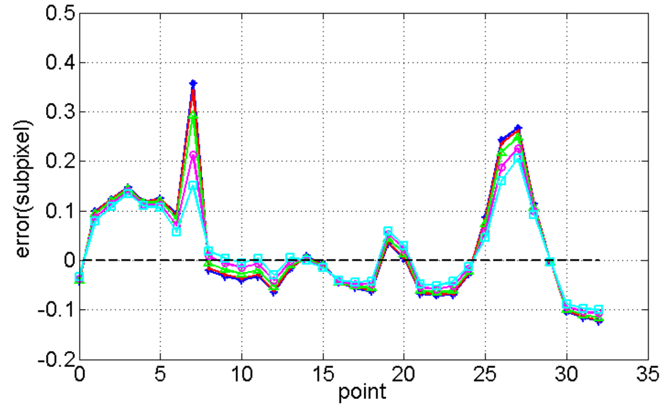
$$C(x,y)=\sum_{k=0}^{m}\sum_{j=0}^{k}a_{kj}x^j y^{k-j} \tag{12}$$

Set error function:

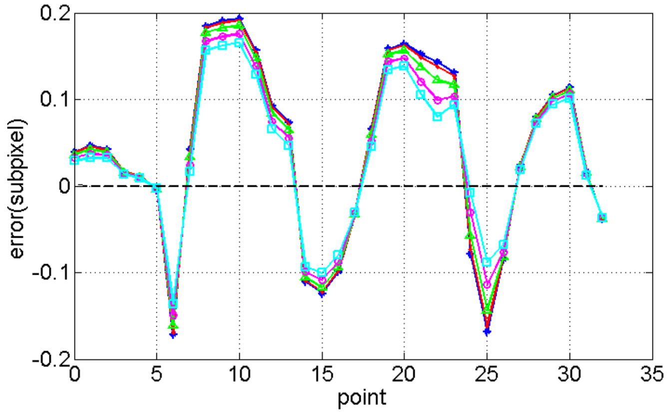
$$\phi(a_{00},a_{01},\cdots,a_{kj})=\sum_{i=1}^{n}(C_i-\sum_{k=0}^{m}\sum_{j=0}^{k}a_{kj}x^j y^{k-j})^2 \tag{13}$$

The cubic surface fitting method needs to use a 5×5 fitting window to calculate, 25 known data points, and 10 undetermined coefficients. From the least squares method, the function $\varphi(a_{00},a_{01},\cdots,a_{33})$ is the minimum, that is, the equations satisfy the condition $\frac{\partial\varphi}{\partial a_{kj}}=0$. Solving the system of equations yields the extreme points of the correlation coefficient $C(x,y)$ and its coordinates (x,y).

We use the surface fitting method and C^n_{ZNCC} of different powers to calculate the sub-pixel displacement of each point through MATLAB, and randomly select 33 points as the error analysis point. Because the above two images only exist integer pixel displacement, the theoretical sub-pixel displacement should be 0. We compare the error of 33 sampling points calculated with the formula A of different powers to the curve of 0 baseline. If the fluctuation of the error curve is smaller and the error value is the smallest, the calculation result of the formula is more stable and accurate.



(a)



(b)

Fig.6 errors of ZNCC with different n

Fig.6 shows the error in the direction of X and Y in the sub pixel displacement of 1 square, 2 square, 5 square, 10 square and 15 square. Fig.6 (a) is the error curve of sub-pixel displacement in the direction of u, and Fig.6 (b) is the error curve of sub-pixel displacement in the direction of v. The blue solid line represents the error curve of the 1 times, the red real line represents the error curve of the 2 square, the green real line indicates the error curve of the 5 square, the pink real line indicates the error curve of the 10 square, the green solid line indicates the error curve of the 15 square, and the black dotted line represents the 0 error line. As can be seen from Fig.6, with the increase of the secondary side, the error is gradually reduced, the error of the 1 square is the largest, the error of the 15 square is the smallest, and the greater the error, the greater the effect of the secondary square number on the error.

Take C_{ZNCC} as an example. When computing the sub-pixel displacement, the average error in the x and y directions, the global average error, and their calculation time when calculating the displacement of the picture with different powers C_{ZNCC} are as follows:

From Table I, it can be seen that the error of the calculation result of the higher power formula is obviously smaller than that of the calculation result of the lower power formula. Moreover, as the number of powers increases, the error and average error in the x and y directions both decrease significantly, and the time for calculating one time is comparable. At the same time, according to the data in Table 1, it can be calculated that the calculated error is reduced by 1.42%; the calculated error is reduced by 6.07%; the calculated error is reduced by 14.17%; the calculated error is reduced by 21.66%. Therefore, we can see that the idea of improving the correlation coefficient formula for this sub-pixel algorithm is obviously optimized.

TABLE I. The Performance of ZNCC With Different n

Formula	Training Set			
	Error/pixel			***Time(s)***
	Error(x)	***Error(y)***	***Average error***	
C_{ZNCC}	0.0881	0.1096	0.0988	3.324384
$C^2{}_{ZNCC}$	0.0869	0.1079	0.0974	3.349788
$C^5{}_{ZNCC}$	0.0829	0.1027	0.0928	3.393246
$C^{10}{}_{ZNCC}$	0.0758	0.0939	0.0848	3.378045
$C^{15}{}_{ZNCC}$	0.0695	0.0854	0.0774	3.370907

5. Conclusion

In this paper, the correlation coefficient is improved, and the correlation coefficient formulas of various correlation coefficients and the improved higher-order square mean-normalized correlation coefficient formulas are used to match the images before and after computer translation, and sub-pixel level displacement calculation is performed. From the correlation coefficient graph, the correlation coefficient formula of zero-mean normalization has the best effect in application. Compared with other correlation coefficient formulas, it can be seen that the peak is sharper, the main peak is more obvious, and the secondary peak is less. The difference between the peaks is large; however, the higher-order zero-mean normalized correlation coefficient formula improved by the improved method is better than the original zero-mean normalized correlation coefficient formula. Within a certain range of n, if the n is larger, the main peak is sharper and more prominent, and the secondary peak is smaller, besides, the difference between the primary and secondary peaks is larger. Even after n takes a certain value, the secondary peak is basically filtered out and can be ignored. Only one single is left, and it is the main peak.

From the calculation results, we can see that in the application of integer-pixel search or calculation, the improved method has no obvious optimization in the global search or point-by-point calculation method, but in the search with a certain step size, the improvement method has obvious optimization. Take a certain value of n so that all secondary peaks are omitted, which can avoid falling into the local maximum peak and increase certain accuracy. In the field of sub-pixel algorithms, the higher-order correlation coefficient formula is obviously optimized for calculating the sub-pixel values using the correlation coefficient values. For example, the errors of surface fitting method in the x and y directions reduce significantly with the n increase. The error of the calculated value of the sub-pixel algorithm gradually decreases, and the error calculated by formula $C^{15}{}_{ZNCC}$ is reduced by 21.66% compared with the error calculated by formula C_{ZNCC}.

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References

[1] ZHANG Shiqiang, LU Jieneng, ZHANG Lei. "Study of the Correlation Coefficients in Mathematical Statistics," Mathematics in Practice and Theory, vol. 39, 2009, pp. 102-107.

[2] WANG Ting, ZHANG Shiqiang. "Study on Linear Correlation Coefficient and Nonlinear Correlation Coefficient in Mathematical Statistics," Studies in Mathematical Sciences, vol.3, 2011,

pp.58-63.

[3] Jan Hauke, Tomasz Kossowski. “COMPARISON OF VALUES OF PEARSON’S AND SPEARMAN’S CORRELATION COEFFICIENTS ON THE SAME SETS OF DATA,” QUAESTIONES GEOGRAPHICAE, vol.32, 2011, pp.87-93 .

[4] Pan B. “Recent Progress in Digital Image Correlatio,”. Experimental Mechanics, vol. 51, 2011, pp.1223-1235.

[5] Christoph Hartmann, Jue Wang. “Daniel Opristescu. Implementation and evaluation of optical flow methods for two-dimensional deformation measurement in comparison to digital image correlation,” Optics and Lasers in Engineering, vol.1, 2018, pp.127-141.

[6] Quan Zhou, Liang Yang, Hui Cao. “A Configurable Circuit for Cross-Correlation in Real-Time Image Matching. Journal of Computer Science and Technology, vol.32, 2017, pp.1305-1318.

[7] Bing Pan, Anand Asundi, Huimin Xie, Jianxin Ga.. “Digital image correlation using iterative least squares and pointwise least squares for displacement field and strain field measurements,” Optics and Lasers in Engineering, vol. 47, 2009, pp.865-874.

[8] YANG Zhou, ZHANG Yimin, ZHANG Xufang, HUANG Xianzhen. “Reliability Sensitivity-based Correlation Coefficient Caculation in Structural Reliability Analysis,” CHINESE JOURNAL OF MECHANICAL ENGNEERING, vol. 25, 2013, pp.608-614.

[9] Zi, Xinyun, Shuai Genghufan Zhao, an, Sd Fang Shu. “Measurement of short shaft power based on a digital speckle correlation method,” Measurement Science and Technology, vol.26, 2015, pp.45001-45008.

[10] Duan W. “Automatic object image alignment using Fourier Descriptors,” Image and Vision computer, vol. 26, 2008, pp.1196-1206.

[11] Xiangru Li, Guanghui Wang, Q.M. Jonath Wu. “Research on the fundamental principles and characteristics of correspondence function,” Mathematical Problems in Engineering, vol.6, 2015, pp.1-15.

[12] W.Tong. “An Evaluation of Digital Image Correlation Criteria for Strain Mapping Applications,” Strain, vol. 41, 2005, pp.167-175.