Partial Relay Selection for Covert Communication in Two-Hop Relay Systems

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This paper investigates the impact of relay selection on covert communication performance in a two-hop wireless relay system, where a transmitter intends to transmit messages to a receiver via a selected relay, subjecting to detection from a warden. To this end, we first propose a partial relay selection scheme based on the signal-to-interference-plus-noise-ratio (SINR) at the relay. We then provide theoretical models for the detection error probability, transmission outage probability, and covert rate. We further explore the optimal designs of the transmit power and target transmission rate for the covert rate, subjecting to the constraints of covertness and the upper bound of transmit power. Finally, we provide extensive numerical results to illustrate the covert rate performance via the partial relay selection scheme in a two-hop relay system.

***Index Terms*—two-hop wireless networks, covert communication, relay selection, covert rate.**

I. Introduction

DUE to the broadcast nature of wireless communication, it faces threats of data interception, eavesdropping or tampering from malicious attackers. Covert communication, also called the low probability of detection (LPD) communication, has recently been recognized as an enhanced secure communication paradigm for wireless communication systems [1], [2]. Compared with the physical layer security (PHY) approach [3], [4], which mainly protects transmitted content against eavesdropping by exploiting the inherent randomness nature of wireless channels, covert communication aims to hide the existence of the transmission process from a malicious detection.

By now, many significant works have been done on exploring the performance of covert communications in different wireless networks, including non-orthogonal multiple access (NOMA) networks [5], relay networks [6], random wireless networks, unmanned aerial vehicle (UAV) networks [7], and others. We note that relay networks play an attractive role in achieving covert communications. Two-hop relay networks can effectively address the problem posed by the limited transmission distance resulting from the low transmit power in covert communication scenarios. On the other hand, relaying in wireless networks can be categorized into transmission modes (i.e., full-duplex (FD) or half-duplex (HD)) and forwarding modes (i.e., amplify-and-forward (AF) or decode-and-forward (DF)), which can be flexibly adapted to practical environments to control the performance of covert communication.

Some works have been conducted on covert communication with one relay in a two-hop wireless system. The work in [8] studied the performance of covert communication in an AF relay network where the relay opportunistically transmits its information to the receiver while forwarding the information from Alice to Bob. Similar to the system in [8], the work in [9] further investigated the impact of outdated channel state information (CSI) on the achievable covert communication performance. The work in [10] explored a transmission and jamming power allocation strategy to satisfy covert and secure requirements in the untrusted AF relaying network. Furthermore, the work in [11] employed the AF relay to send jamming, which can improve the instantaneous and average covert rate when the system works solely under FD mode or HD mode. Considering a DF relay in the two-hop system, the works in [12] and [13] suggested different beamforming schemes for covert communication with a multi-antenna relay.

For multiple-relay networks, relay selection has been regarded as an effective technique to realize enhanced covert communication. Covert communication with relay selection is first explored in [14]. The work in [15] proposed random and superior link selection strategies in a multiple relay system, where the covert capacity under the superior link selection scheme is always higher than that under the random selection scheme with the fixed parameters of the considered system. The works in [16] and [17] investigated covert communication where the relay and the jammer are jointly selected. In [16], only one jammer with minimal channel gain to the receiver is selected to send jamming signals to prevent the detector from detecting the covert transmission. In [17], multiple jammers with channel gain to the receiver smaller than a given threshold are selected to send jamming signals to prevent the detector from detecting the covert transmission. Although the above works have been studied to help us understand the design of covert communication schemes with relay selection, these works focus on DF relay-assisted in multiple-relay networks. We note that no work has explored the covert performance of the AF relay system. Thus, this article aims to study the performance of covert communication with relay selection in the AF relay system. The main contributions of this article are summarized as follows.

- We consider a two-hop wireless system consisting of one

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transmitter Alice, several AF relays, one receiver Bob and one Warden Willie. In this scenario, Alice selects one relay through a partial relay selection scheme to help forward the covert message to Bob. To achieve covertness, the FD Alice emits the jamming signal to confuse Willie.

- Under the partial relay selection scheme, we first derive the expressions for the minimum detection error probability with optimal detection threshold and determine its average value as the probability that the warden detects the transmission. We then provide theoretical modeling for the transmission outage probability to depict the probability that messages cannot be transmitted reliably. We further explore the covert rate by providing an optimization problem formulation to identify the optimal designs of the transmitter transmission rate and the transmit power, subjecting to the constraints of detection error probability and an upper bound on transmit power.
- We provide extensive simulations and numerical results to validate our theoretical analysis and illustrate the covert performance of the relay system with the settings of transmitter transmission rate, upper bound on transmit power and constraint of detection error probability.

The rest of this paper is organized as follows. Section II introduces the system model and performance metrics. Section III presents the performance analysis of covert communication. The simulation and numerical results are illustrated in Section IV. Finally, Section V concludes this work.

II. System Model and Performance Metrics

A. Communication Scenario and Assumptions

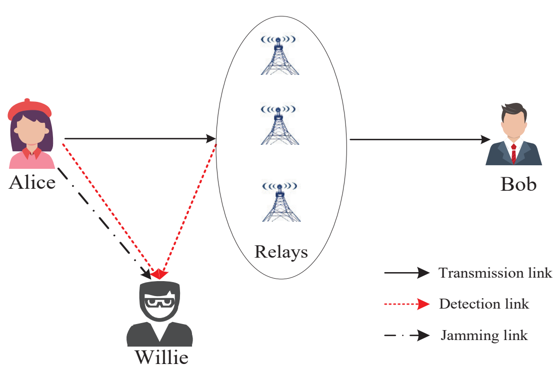

Fig. 1. System model

As illustrated in Fig. 1, we consider a two-hop wireless system consisting of one transmitter (Alice), M amplify-and-forward (AF) relays (Relays) denoted by $R = \{R_k | k = 1, 2, ..., K\}$, one receiver (Bob) and one warden (Willie). We assume that Bob and Willie each have a single antenna while Alice and Relay equip a pair of transmit-receive antennas, so Bob and Willie operate in HD mode while Alice and Relay operate in FD mode. Alice tries to send covert messages to Bob with the help of one relay (i.e., the optimal relay selected by the selection criteria in the next section) in the presence of Willie, who aims to detect whether Alice is transmitting messages to Bob or not. We assume that Alice uses a secret Gaussian codebook pre-shared with Bob, but unknown to Willie [18]. Alice sends the covert signal to R_k in one random time slot and emits the jamming signal in all time slots to confuse Willie's detection.

We consider a time-slotted quasi-static Rayleigh fading channel model in our system, where the channel coefficient remains constant in one slot and changes independently and randomly from one slot to another. The channel coefficient from node I to J is denoted as h_{IJ} which is a complex zero-mean random variable with variance $\mathbb{E}[|h_{IJ}|^2] = \lambda_{IJ}$, and the probability density function (pdf) of $|h_{IJ}|^2$ is given by $f_{|h_{IJ}|^2} = (1/\lambda_{IJ})\exp(-x/\lambda_{IJ})$. Here, $IJ \in \{ar_k, r_kb, aw, r_kw\}$ where a, r_k, b and w represent Alice, Relay, Bob, and Willie, respectively. Also, after the imperfect self-interference cancellation scheme, the residual self-interference channel at Relay is assumed to be subject to independent and identically distributed Rayleigh fading with variance $E[|h_{r_kr_k}|^2] = \lambda_{r_kr_k}$. Considering channel reciprocity, we assume the channel coefficient from Alice to Relay is the same as that from Relay to Alice, i.e., $h_{ar_k} = h_{r_ka}$. Alice sends pilot signals to estimate the channel state information (CSI) from Alice to Relay, and Relay also sends pilot signals to estimate the CSI from Relay to Bob. Thus, Alice and Bob know instantaneous channel coefficients h_{ar_k} and h_{r_kb}, respectively. Relay knows both h_{ar_k} and h_{r_kb}, while Willie also knows the h_{aw} and h_{r_kw}. Thus, the availability of h_{aw} and h_{r_kw} at Willie represents the worst-case scenario from the perspective of covert communication design.

B. Transmission Process

We assume that Alice equips two antennas, one of which is used to send the covert signal and the other to send the jamming signal in order to assist the covert communication. When Alice selects one time slot to transmit a covert signal, Alice first encodes the signal into n symbols $\mathbf{X}_a = \{x_a[i]\}_{i=1}^n$, where each symbol $x_a[i]$ is subject to the constraint of $E[|x[i]|^2] = 1$. $E[\cdot]$ represents the expectation operator. Then Alice transmits the symbols to Relay with a transmit power P in one time slot. To confuse Willie's detection, Alice sends a jamming signal with a transmit power P_j following a continuous uniform distribution over the interval $[0, P_j^{\max}]$, having a probability density function (pdf) given by

$$f_{P_j}(x) = \begin{cases} \frac{1}{P_j^{\max}}, & 0 \le P_j \le P_j^{\max} \\ 0, & otherwise. \end{cases} \quad (1)$$

We assume Relay and Bob can cancel out the effect of jamming signal [19], [20], while Relay suffers the self-interference in the FD mode. Thus, the received symbol for the i-th channel use at Relay is given by

$$y_{r_k}[i] = \sqrt{P}h_{ar_k}x_a[i] + \sqrt{P}h_{r_kr_k}x_{r_k}[i] + n_{r_k}[i], \quad (2)$$

where $x_{r_k}[i]$ is the transmitted symbol from Relay, which is given by $x_{r_k}[i] = \beta y_{r_k}[i]$, where the amplification factor β is $\beta = \sqrt{\frac{1}{P|h_{ar_k}|^2 + P|h_{r_kr_k}|^2 + \sigma_{r_k}^2}}$; $n_{r_k}[i]$ is the AWGN at Relay with zero mean and variance $\sigma_{r_k}^2$, i.e., $n_{r_k}[i] \sim \mathcal{CN}(0, \sigma_{r_k}^2)$.

Relay amplifies and forwards its received symbol to Bob, so the symbol received at Bob in the i-th channel use is given by

$$y_b[i] = \sqrt{P} h_{r_k b} x_{r_k}[i] + n_b[i]. \tag{3}$$

where $n_b[i]$ is the AWGN at Bob with zero mean and variance σ_b^2, i.e., $n_b[i] \sim \mathcal{CN}(0, \sigma_b^2)$.

C. Detection at Willie

As in [2], Willie is the detector to decide whether Alice transmits covert messages or not. Willie conducts binary hypothesis testing, where the null hypothesis H_0 denotes that Alice does not transmit covert messages. In contrast, the alternative hypothesis H_1 denotes that Alice transmits. When H_0 is true, Willie receives only the jamming signal from Alice. When H_1 is true, Willie receives the signals (i.e., covert and jamming signals) from Alice and the amplified signal from Relay. Thus, the received signals at Willie are given by

$$\mathbf{y}_w[i] = \begin{cases} \sqrt{P_j} h_{aw} x_j[i] + n_w[i], & H_0, \\ \phi + \sqrt{P_j} h_{aw} x_j[i] + n_w[i], & H_1, \end{cases} \tag{4}$$

where $\phi = \sqrt{P} h_{aw} x_a[i] + \sqrt{P} h_{r_k w} x_{r_k}[i]$; $n_w[i]$ is the AWGN at Willie with zero mean and variance σ_w^2, i.e., $n_w[i] \sim \mathcal{CN}(0, \sigma_w^2)$.

Willie adopts a radiometer to make the decision on whether covert communication exists or not by evaluating the total received power in one time slot. Thus, Willie conducts a threshold test on the average power received, given by

$$\Omega \triangleq \frac{1}{n} \sum_{i=1}^{n} |y_w[i]|^2 \underset{D_0}{\overset{D_1}{\gtrless}} \tau, \tag{5}$$

where Ω is the average power of received signals and τ is the detection threshold for Willie's test; D_0 and D_1 are the binary decisions that denote Willie makes a decision in favor of H_0 and H_1, respectively. We define one detection error, called false alarm (FA), which means that Willie's decision is D_1 while H_0 is true. Another error is called missed detection (MD), which means that Willie's decision is D_0 while H_1 is true. The probabilities of FA and MD are $p_{FA} = \mathbb{P}(D_1|H_0)$ and $p_{MD} = \mathbb{P}(D_0|H_1)$, respectively. Since Willie has no information on when Alice transmits, it considers the prior probabilities of hypotheses $\mathbb{P}(H_0)$ and $\mathbb{P}(H_1)$ are equal. Then, the detection error probability is given by

$$\xi = p_{FA} + p_{MD}. \tag{6}$$

As such, the covert constraint is then set as $\xi \geq 1-\epsilon$, where ϵ is an arbitrarily small constant denoting the covert requirement. We say that covert communication can be achieved when the detection error probability satisfies the covertness requirement.

D. Performance Metrics

From Alice's point of view, if the target transmission rate R_{ab} between Alice and Bob is greater than the channel capacity C, Alice cannot connect to Bob, i.e., the transmission outage happens in the sense that Bob cannot recover the messages reliably [8]. Let p_{out} denote the **transmission outage probability**, which is given by

$$p_{out} = \mathbb{P}\{C < R_{ab}\}. \tag{7}$$

We introduce **covert rate** to depict the performance of the covert communication system with the partial relay selection scheme. The covert rate is defined as the achievable rate of successfully transmitted covert messages from Alice to Bob subject to the covertness requirement $\xi \geq 1-\epsilon$ and maximum cover signal transmit power of Alice $P^{\max}$ with the given upper bound of jamming signal transmit power $P_j^{\max}$, which is given by

$$R_c = \quad R_{ab}\,(\,1 - p_{out}\,), \tag{8a}$$
$$\text{s.t.} \quad \xi \geq 1 - \epsilon \tag{8b}$$
$$0 \leq P \leq P^{\max} \tag{8c}$$

III. Performance Analysis of Covert Communication

This section investigates the covert rate under the partial relay selection scheme. We see from (8) that detection error probability and transmission outage probability can determine the covert rate. We first derive the detection error probability and transmission outage probability in Subsections II-A and II-B, respectively. We further explore the covert rate by optimizing Alice's transmission rate and covert signal transmit power with a given upper bound of jamming signal transmit power.

A. Detection Error Probability

Based on the detection scheme in Subsection II-C, Willie decides on the existence of transmitted covert symbols in one time slot by comparing the average power received to a detection threshold. When Alice does not transmit covert symbols, Willie accepts the $\mathcal{D}_1$ by $\Omega \geq \tau$, leading to a false alarm. Thus, the false alarm probability p_{FA} is determined as

$$\begin{aligned} p_{FA} &= \mathbb{P}\{(P_j|h_{aw}|^2 + \sigma_w^2)\frac{\chi_{2n}^2}{n} \geq \tau \,|\, H_0\} \\ &= \mathbb{P}\{(P_j|h_{aw}|^2 + \sigma_w^2) \geq \tau \,|\, H_0\} \\ &= \begin{cases} 1, & \tau \leq \sigma_w^2, \\ 1 - \frac{\tau - \sigma_w^2}{P_j^{\max}|h_{aw}|^2}, & \sigma_w^2 < \tau \leq \mu, \\ 0, & \tau > \mu, \end{cases} \end{aligned} \tag{9}$$

where $\mu = P_j^{\max}|h_{aw}|^2 + \sigma_w^2$; χ_{2n}^2 is a chi-squared random variable with $2n$ degrees of freedom. Based on the Strong Law of Large Numbers and Lebesgue's Dominated Convergence Theorem [21], $\frac{\chi_{2n}^2}{n}$ converges in probability to 1 as n tends to infinity. When Alice transmits, Willie accepts $\mathcal{D}_0$ by $\Omega < \tau$

in this case, leading to a missed detection. Thus, the missed detection probability p_{MD} is determined as

$$p_{MD} = \mathbb{P}\{(P|h_{aw}|^2 + P|h_{r_kw}|^2 + P_j|h_{aw}|^2 + \sigma_w^2) < \tau \mid \mathcal{H}_0\} = \begin{cases} 0, & \tau \le \nu, \\ \frac{\tau-\nu}{P_j^{\max}|h_{aw}|^2}, & \nu < \tau \le \nu + P_j^{\max}|h_{aw}|^2, \\ 1, & \tau > \nu + P_j^{\max}|h_{aw}|^2. \end{cases} \quad (10)$$

where $\nu = P|h_{aw}|^2 + P|h_{r_kw}|^2 + \sigma_w^2$.

Substituting p_{FA} in (9) and p_{MD} in (10) into ξ in (6), we can obtain the detection error probability ξ at Willie. We note that the value of μ in p_{FA} and ν in p_{MD} brings two cases (i.e., $\mu \le \nu$ and $\mu > \nu$) to determine the ξ. when $\mu \le \nu$, the ξ is determined as

$$\xi(\tau) = \begin{cases} 1, & \tau \le \sigma_w^2, \\ 1 - \frac{\tau-\sigma_w^2}{P_j^{\max}|h_{aw}|^2}, & \sigma_w^2 < \tau \le \mu, \\ 0, & \mu < \tau \le \nu, \\ \frac{\tau-\nu}{P_j^{\max}|h_{aw}|^2}, & \nu < \tau \le \nu + P_j^{\max}|h_{aw}|^2, \\ 1, & \nu + P_j^{\max}|h_{aw}|^2 < \tau. \end{cases} \quad (11)$$

when $\mu > \nu$, the ξ is determined as

$$\xi(\tau) = \begin{cases} 1, & \tau \le \sigma_w^2, \\ 1 - \frac{\tau-\sigma_w^2}{P_j^{\max}|h_{aw}|^2}, & \sigma_w^2 < \tau \le \nu, \\ 1 - \frac{\nu-\sigma_w^2}{P_j^{\max}|h_{aw}|^2}, & \nu < \tau \le \mu, \\ \frac{\tau-\nu}{P_j^{\max}|h_{aw}|^2}, & \mu < \tau \le \nu + P_j^{\max}|h_{aw}|^2, \\ 1, & \nu + P_j^{\max}|h_{aw}|^2 < \tau. \end{cases} \quad (12)$$

We can see from (11) that ξ decreases as τ increases when $\tau \in (\sigma_w^2, \mu]$ and ξ increases as τ increases when $\tau \in (\nu, \nu + P_j^{\max}|h_{aw}|^2]$. Thus, the minimum detection error probability $\xi^* = 0$ with the optimal detection threshold $\tau^* \in [\mu, \nu]$. Following (12), when $\tau \in (\sigma_w^2, \nu]$, we have $\frac{\partial \xi(\tau)}{\partial \tau} < 0$ which indicates that ξ decreases with τ. when $\tau \in (\nu, \nu + P_j^{\max}|h_{aw}|^2]$, we have $\frac{\partial \xi(\tau)}{\partial \tau} > 0$ which indicates that ξ increased with τ. Thus, the minimum detection error probability $\xi^* = 1 - \frac{P|h_{aw}|^2 + P|h_{r_kw}|^2}{P_j^{\max}|h_{aw}|^2}$ with $\tau^* \in [\sigma_w^2, \mu]$. Thus, we have

$$\xi^* = \begin{cases} 0, & \mu \le \nu, \\ 1 - \frac{P|h_{aw}|^2 + P|h_{r_kw}|^2}{P_j^{\max}|h_{aw}|^2}, & \mu > \nu. \end{cases} \quad (13)$$

Since Alice and Relay do not know the instantaneous channel coefficient h_{aw} and h_{r_kw}, we consider the expected value of minimum detection error probability at Willie. Let ξ^* denote the average minimum detection error probability, which is given in the following theorem.

Theorem 1. *Consider the covert communication in a relay-assisted system where Alice and Relay transmit the covert signal with transmit power P and Alice sends the jamming signal with transmit power P_j following a continuous uniform distribution over the interval $[0, P_j^{\max}]$. If the variance of the CSI h_{aw} and h_{r_kw} are denoted as λ_{aw} and λ_{r_kw}, the average minimum detection error probability $\overline{\xi^*}$ is determined as*

$$\overline{\xi^*} = \frac{(P_j^{\max} - P)\lambda_{aw}}{(P_j^{\max} - P)\lambda_{aw} + P\lambda_{r_kw}} - \frac{(P_j^{\max} - P)^2 P(\lambda_{aw} - \lambda_{r_kw})\lambda_{aw}}{P_j^{\max}[(P_j^{\max} - P)\lambda_{aw} + P\lambda_{r_kw}]^2} - \frac{(P_j^{\max} - P)P\lambda_{r_kw}}{P_j^{\max}[(P_j^{\max} - P)\lambda_{aw} + P\lambda_{r_kw}]}\ln(1 + \frac{(P_j^{\max} - P)\lambda_{aw}}{P\lambda_{r_kw}}). \quad (14)$$

Proof. We know that $|h_{aw}|^2$ and $|h_{r_kw}|^2$ follows exponential distributions with expected values λ_{aw} and λ_{r_kw}. Based on (13), we have

$$\begin{aligned} \overline{\xi^*} =& \mathbb{E}[\xi^*] \\ =& \mathbb{P}\{\mu \le \nu\}\mathbb{E}[0|\mu \le \nu] \\ &+ \mathbb{P}\{\mu > \nu\}\mathbb{E}[1 - \frac{P|h_{aw}|^2 + P|h_{r_kw}|^2}{P_j^{\max}|h_{aw}|^2}|\mu > \nu] \\ =& \mathbb{P}\{\mu > \nu\}\mathbb{E}[1 - \frac{P|h_{aw}|^2 + P|h_{r_kw}|^2}{P_j^{\max}|h_{aw}|^2}|\mu > \nu], \end{aligned} \quad (15)$$

where

$$\begin{aligned} \mathbb{P}\{\mu > \nu\} &= \mathbb{P}\{P_j^{\max}|h_{aw}|^2 > P|h_{aw}|^2 + P|h_{r_kw}|^2\} \\ &= \int_0^\infty \int_0^{\frac{(P_j^{\max} - P)y}{P}} f_{|h_{r_kw}|^2}(x) f_{|h_{aw}|^2}(y) dx dy \\ &= \frac{(P_j^{\max} - P)\lambda_{aw}}{(P_j^{\max} - P)\lambda_{aw} + P\lambda_{r_kw}} \end{aligned} \quad (16)$$

and

$$\begin{aligned} &\mathbb{E}[1 - \frac{P|h_{aw}|^2 + P|h_{r_kw}|^2}{P_j^{\max}|h_{aw}|^2}|\mu > \nu] \\ =& 1 - \mathbb{E}[\frac{P|h_{aw}|^2 + P|h_{r_kw}|^2}{P_j^{\max}|h_{aw}|^2}|\mu > \nu] \\ =& 1 - \int_0^\infty \int_0^{\frac{(P_j^{\max} - P)y}{P}} \frac{Py + Px}{P_j^{\max}} f_{|h_{r_kw}|^2}(x) f_{|h_{aw}|^2}(y) dx dy \\ =& 1 - \frac{P(P_j^{\max} - P)(\lambda_{aw} - \lambda_{r_kw})}{P_j^{\max}[(P_j^{\max} - P)\lambda_{aw} + P\lambda_{r_kw}]} \\ &- \frac{P\lambda_{r_kw}}{P_j^{\max}\lambda_{aw}} \ln\left(1 + \frac{(P_j^{\max} - P)\lambda_{aw}}{P\lambda_{r_kw}}\right). \end{aligned} \quad (17)$$

□

Substituting (16) and (17) into (15), we can obtain (14).

B. Transmission Outage Probability

Based on the received signal at Relay in (2) and the received signal at Bob in (3), the channel capacity C_{r_k} with the k-th Relay is determined as

$$C_{r_k} = \log_2(1 + \frac{\frac{\gamma_{ar_k}}{\gamma_{r_kr_k}+1}\gamma_{r_kb}}{\frac{\gamma_{ar_k}}{\gamma_{r_kr_k}+1} + \gamma_{r_kb} + 1}), \quad (18)$$

where $\gamma_{ar_k} = \frac{P|h_{ar_k}|^2}{\sigma_{r_k}^2}$, $\gamma_{r_kr_k} = \frac{P|h_{r_kr_k}|^2}{\sigma_{r_k}^2}$ and $\gamma_{r_kb} = \frac{P|h_{r_kb}|^2}{\sigma_b^2}$ denote the signal-to-interference-plus-noise-ratio (SINR) of $a \to r_k$, $r_k \to r_k$, $r_k \to b$. From Alice's perspective, a transmission outage occurs when the target transmission rate R_{ab} is greater than the channel capacity C_{r_k}. Substituting (18) into (7), the transmission outage probability p_{out} is written as

$$p_{out} = \mathbb{P}\left\{ \frac{\frac{\gamma_{ar_k}}{\gamma_{r_kr_k}+1}\gamma_{r_kb}}{\frac{\gamma_{ar_k}}{\gamma_{r_kr_k}+1} + \gamma_{r_kb} + 1} < \gamma_{th} \right\}, \tag{19}$$

where $\gamma_{th} = 2^{R_{ab}} - 1$.

In this FD-AF relay scenario, we propose one partial relay selection policy. This policy selects the optimal relay with the maximum ratio between the SINR of $a \to r_k$ and $r_k \to r_k$. Hence, we select the relay where

$$k^* = \arg\max_k \left\{ \frac{\gamma_{ar_k}}{\gamma_{r_kr_k}+1} \right\}, \tag{20}$$

and thus we have

$$\gamma_{k^*} = \max\left\{ \frac{\gamma_{ar_k}}{\gamma_{r_kr_k}+1} \right\}. \tag{21}$$

Here, we continue to derive the transmission outage probability of the system with a partial relay selection policy. Then, the p_{out} can be given by

$$\begin{aligned} p_{out} &= F(\gamma_{th}) \\ &= F_{\gamma_{r_kb}}(\gamma_{th}) + \int_{\gamma_{th}}^{\infty} F_{\gamma_{k^*}}\left(\frac{(y+1)\gamma_{th}}{y-\gamma_{th}}\right) f_{\gamma_{r_kb}}(y)dy \end{aligned} \tag{22}$$

We use a simple order statistic result in [22], [23] to obtain the cumulative distribution function (CDF) of γ_k^*. Since γ_k^* (K-th order statistic) is selected as the maximum of the K Alice-to-Relay link SINRs, $\gamma_k = \frac{\gamma_{ar_k}}{\gamma_{r_kr_k}+1}$, the CDF of γ_k^* is determined as

$$\begin{aligned} F_{\gamma_{k^*}}(x) &= \prod_{k=1}^{K} \mathbb{P}\{\frac{\gamma_{ar_k}}{\gamma_{r_kr_k}+1} < x\} \\ &= \left(\int_0^\infty \int_0^{(t+1)x} f_{\gamma_{ar_k}}(s) f_{\gamma_{r_kr_k}}(t) ds dt \right)^K \\ &= \left(1 - \frac{e^{-\frac{\sigma_{r_k}^2}{P\lambda_{ar_k}}x}}{1 + \frac{\lambda_{r_kr_k}}{\lambda_{ar_k}}x} \right)^K \end{aligned} \tag{23}$$

Substituting (23) into (22), the p_{out} can be written as

$$\begin{aligned} p_{out} =& 1 - e^{-\frac{\sigma_b^2\gamma_{th}}{P\lambda_{r_kb}}} + \frac{\sigma_b^2}{P\lambda_{r_kb}} e^{-\frac{\sigma_b^2\gamma_{th}}{P\lambda_{r_kb}}} \\ &\times \int_0^\infty \left[1 - \frac{e^{-\frac{\sigma_{r_k}^2(z+1+\gamma_{th})\gamma_{th}}{P\lambda_{ar_k}z}}}{1 + \frac{\lambda_{r_kr_k}(z+1+\gamma_{th})\gamma_{th}}{\lambda_{ar_k}z}} \right]^K e^{\frac{\sigma_b^2 z}{P\lambda_{r_kb}}} dz \end{aligned} \tag{24}$$

where $z = y - \gamma_{th}$.

C. Covert Rate

Our objective is to obtain the maximum covert rate R_c, subject to the covertness constraint on $\overline{\xi^*}$ and maximum power constraint on Alice's covert signal transmit power P. Thus, R_c is formulated as

$$\max_{P,R_{ab}} \quad R_c, \tag{25a}$$

$$\text{s.t.} \quad \overline{\xi^*} \geq 1 - \epsilon \tag{25b}$$

$$0 \leq P \leq P^{\max} \tag{25c}$$

Note that the optimization problem in (25) involves two performance metrics (i.e., average minimum detection error rate and transmission outage probability). The expressions of these metrics are too complex to obtain the closed-form solutions of the optimization problems. Thus, we can apply a traversal searching algorithm in Algorithm 1 to solve the optimization problem in (25).

Algorithm 1: Covert Rate Traversal Searching Algorithm

Input: Maximum covert signal transmit power $P^{\max}$, covert requirement ϵ, maximum jamming signal transmit power $P_j^{\max}$, average channel gain of self-interference $\lambda_{r_kr_k}$;

Output: Covert rate R_c, the corresponding optimal covert signal transmit power P^* and target transmission rate R_{ab}^*;

Initialize set the length L_P for P, L_R for R_{ab}, $P = 0$, and $R_c^1 = R_c^0 = 0$, Target transmission rate $R_{ab} = 0$, the iteration index $\eta = 0$ and the maximum number of iterations $\eta^{\max}$, $\overline{\xi^*} = 1$;
for $\eta = 1$; $\eta \leq \eta^{\max}$; $\eta++$ **do**
  if $P \leq P^{\max}$ and $\overline{\xi^*} \geq 1 - \epsilon$ **then**
    $P = P + L_P$;
    Calculate $\overline{\xi^*}$ according to (14) and obtain the P^*;
  end
  Calculate $R_c(P^*, R_{ab})$ according to (25a);
  if $R_c^{\eta}(P^*, R_{ab}) \geq R_c^{\eta-1}(P^*, R_{ab})$ **then**
    $R_{ab} = R_{ab} + L_R$;
    Calculate $R_c(P^*, R_{ab})$ according to (25a) and obtain the R_{ab}^*;
  end
end
return R_c, P^*, R_{ab}^*;

IV. Numerical results

In this section, we provide extensive numerical results to illustrate the performance of our proposed partial relay selection scheme in terms of average minimum detection error probability $\overline{\xi^*}$, transmission outage probability p_{out}, and covert rate R_c. The case of $K = 1$ corresponds to no relay selection, which is also depicted as a benchmark for comparison. Unless otherwise stated, in the numerical results, the related parameters are set as $P^{\max} = 1$W and $\lambda_{aw} = \lambda_{r_kw}$ and $\sigma_{r_k}^2 = \sigma_b^2 = \sigma_w^2 = 10^{-4}$W.

A. Performance Analysis of Detection Error Probability

To explore the impact of covert signal transmit power P on average minimum detection error probability $\overline{\xi^*}$, we summarize in Fig. 2 how $\overline{\xi^*}$ vary with P for a setting of $P_j^{\max} = \{10, 20, 30\}$W. We can see from Fig. 2 that as P increases, $\overline{\xi^*}$ decreases. This is due to the fact that when the $P_j^{\max}$ is fixed, Willie becomes easier to detect the covert communication between Alice and Bob with increasing P. We can also see from Fig. 2 that $\overline{\xi^*}$ increases as $P_j^{\max}$ increases with a fixed P. This is because when $P_j^{\max}$ is larger, the effect of the jamming signal becomes larger, leading to a higher detection error probability at Willie.

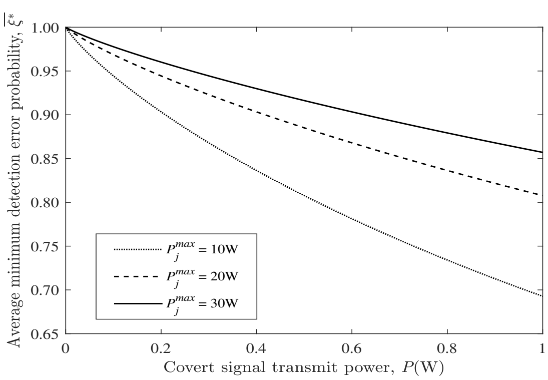

Fig. 2. Average minimum detection error probability $\overline{\xi^*}$ vs. covert signal transmit power P.

B. Performance Analysis of Transmission Outage Probability

We first show in Fig. 3 the impact of covert signal transmit power P on the transmission outage probability p_{out} under the settings of $P_j^{\max} = 30$W, $R_{ab} = 8$ bits per channel use, $\epsilon = 0.1$ and $\lambda_{r_k r_k} = 0.1$. We can observe from Fig. 3 that as P increases, p_{out} decreases. The reason is that when P increases, the SINR between Alice and Bob increases, leading to the p_{out} decreases. We further observe from Fig. 3 that p_{out} increases as K. This proves that the increasing number of relays can lead to the increasing channel gain from Alice to Bob, and thus a smaller p_{out}.

We then show in Fig. 4 the impact of target transmission rate R_{ab} on the transmission outage probability p_{out} under the settings of $P_j^{\max} = 30$W, $\epsilon = 0.1$, $\lambda_{r_k r_k} = 0.1$ and $R_{ab} = 8$ bits per channel use. It can be seen from Fig. 3 that when $\lambda_{r_k r_k}$ is relatively small (e.g., $\lambda_{r_k r_k} = 0.1$), as R_{ab} increases, p_{out} first increases and then becomes 1. The reason can be explained as follows. When R_{ab} becomes larger, Bob cannot decode all the received signals reliably, leading to the increase of the p_{out}. We also see from Fig. 4 that for a fixed R_{ab}, p_{out} decreases as K increases under the partial relay selection scheme. The reasons behind those observations are similar to those illustrated in Fig. 3.

C. Covert Rate

We investigate the impact of covert signal transmit power P on the covert rate R_c, and summarize in Fig. 5 how the covert rate R_c varies with covert signal transmit power P under the settings of $P_j^{\max} = 30$W, $\epsilon = 0.1$, $K = 2$, $P^{\max} = 1$W and $R_{ab} = 10$ bits per channel use. We can see from Fig. 5 that R_c first increases with P and then remains unchanged. This is because for the fixed $\epsilon = 0.1$, P increases, SINRs at Relay and Bob become larger, leading to R_c increases. Since the communication should meet the covert requirement, the R_c remains unchanged when P increases further. We can also see from Fig. 5 that as λ_{rr} increases, R_c decreases. This proves that the performance of R_c in the system is greatly affected by the self-interference from the FD relay.

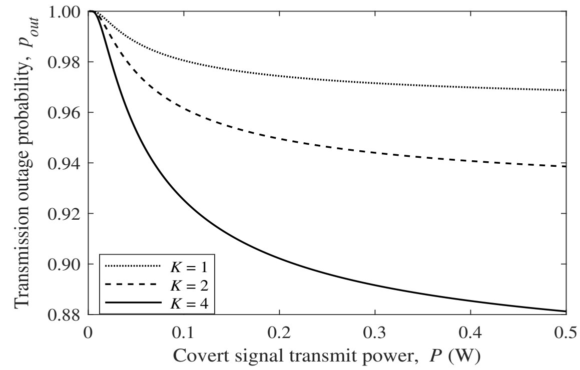

Fig. 3. Transmission outage probability p_{out} vs. covert signal transmit power P.

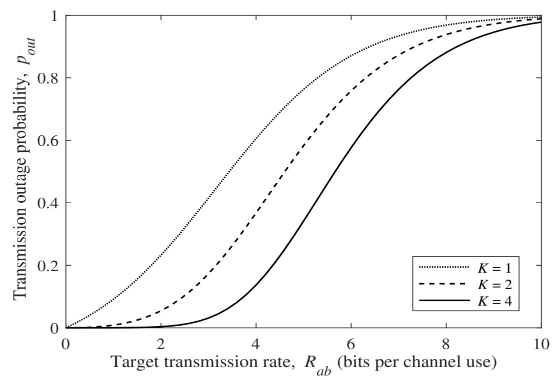

Fig. 4. Transmission outage probability p_{out} vs. target transmission rate R_{ab}.

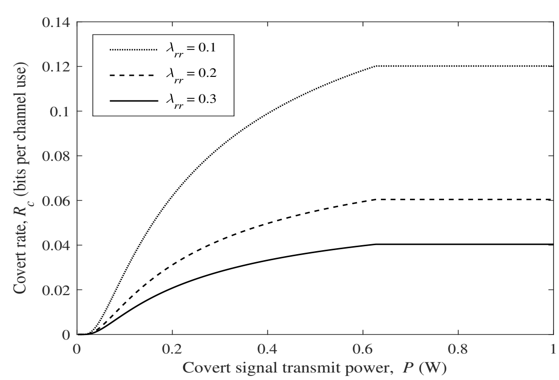

Fig. 5. Covert rate R_c vs. covert signal transmit power P.

We then investigate the impact of the target transmission rate R_{ab} on the covert rate R_c, and summarize in Fig. 6 how the covert rate R_c varies with target transmission rate R_{ab} under the settings of $P_j^{\max} = 30$W, $\epsilon = 0.1$, $P = 0.3$W. We can see from Fig. 6 that as R_{ab} increases, R_c first increases, then achieves a maximum value and decreases. This means that when Alice sets a proper R_{ab} with an optimal P, we can obtain a maximum R_c. When R_{ab} increases further, the R_{ab} becomes larger than the channel capacity, leading to a larger p_{out} and thus a smaller R_c. Another observation from Fig. 6 indicates that for a fixed R_{ab}, R_c increases as K increases. The reasons behind the observations are similar to those illustrated in Fig. 3.

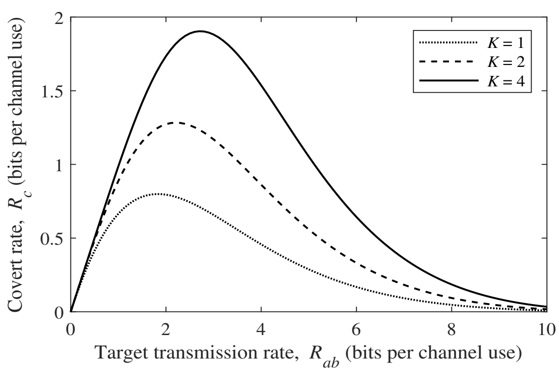

Fig. 6. Covert rate R_c vs. target transmission rate R_{ab}.

V. Conclusion

This paper explored the impact of the proposed partial relay selection scheme on covert communication performance in the two-hop wireless relay system. We developed the theoretical modeling in terms of the average minimum detection error probability, transmission outage probability, and covert rate. We further explored the optimal target rate and transmit power to achieve the maximum covert rate, subjecting to the constraints of covertness and transmit power. Comparing to the single relay system as the benchmark, our results demonstrated that the relay selection technique in a multiple relay system can enhance the performance of covert communication.

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