On Covert Communication Performance in Multi-relay Systems with Outdated CSI

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Communication performance significantly depends on the availability of Channel State Information (CSI). This paper investigates the impact of outdated CSI on achievable covert communication performance in a multiple-relay system under a power control scheme. Considering the feedback delay of all links, we first develop an accurate characterization of outdated CSI. Based on this, we formulate theoretical models to depict the inherent relationship between outdated CSI and fundamental covert performance metrics, namely detection error probability (DEP) and covert rate (CR). Utilizing these models, we delve into optimizing CR subject to the DEP constraint, unveiling the maximal CR performance achievable with outdated CSI. Extensive numerical results illustrate the impact of outdated CSI on covert communication performance, also indicating that an increase in the number of relays can mitigate the negative effects of outdated CSI.

Index Terms—**Covert Communication, Outdated CSI, Multiple-relay, Detection Error Probability, Maximal CR.**

I. INTRODUCTION

Wireless communication, evolving rapidly towards the 5G/6G era, is set to integrate the digital and physical worlds more closely, leading to the widespread transmission of sensitive data. However, the inherent openness of wireless networks makes them vulnerable to security threats, a concern in both civilian and military domains. As we advance into the 5G/6G era, developing robust security solutions to protect the privacy and integrity of data in these networks becomes imperative, marking a crucial step in ensuring secure and private communication in an increasingly connected world [1].

Covert communication, a new security paradigm that can protect the transmission process, is hoping to become an essential complement to traditional cryptography-based and physical layer security [2]. In the pioneering work, Bash et al. established a square root scaling law for covert communication on the additive white Gaussian noise (AWGN) channel, stating that in the use of n channels, no more than $O(\sqrt{n})$ bits of information can be reliably transmitted from a transmitter to a receiver while keeping covertly from the warden [3]. Then many studies further examined information-theoretic limits of covert communication under other channels, such as the Binary Symmetric Channels (BSCs) [4], Discrete Memoryless Channels (DMCs) [5], Multiple Access Channels (MACs) [6], and continuous-time Poisson channels [7].

Recently, the application scenarios for covert communication have become incredibly diverse. These include exploiting Non-Orthogonal Multiple Access (NOMA) multi-user characteristics to hide a covert user's signals within those of a legitimate user [9], [40]; utilizing the mobility of Unmanned Aerial Vehicles (UAVs) for improved network configuration and enhanced transmission rates through Line-of-Sight (LoS) channels [10]–[13]; integrating Device-to-Device (D2D) communication to enable direct device interactions, bypassing a base station and thereby improving covert capabilities [14]–[16]; employing Intelligent Reflecting Surfaces (IRS) (a.k.a. Reconfigurable Intelligent Surfaces (RIS)) to optimize signal quality at the receiver side while impairing detection by the warden [17]; combining Multiple Input Multiple Output (MIMO) and beamforming technology to increase covert throughput [18]–[20]; and using two-hop relay systems to extend the range of covert communication, which adopts lower transmission power for covert signals, leading to shorter transmission distances for single-hop transmissions [23]–[31]. Here, we focus on the scenario of two-hop relay systems, as they are building blocks for more complex covert networks, such as multi-hop [21], [22], Internet of Things (IoT) [32], and satellite covert networks [33].

Specifically, the authors in [23] explored a scenario involving a greedy relay, which, while forwarding information from the source node, also opportunistically transmits its own covert messages to the destination node. In this case, the source node assumes the role of a detector. Subsequently, the authors in [24] extended this concept by equipping such a greedy relay with multiple antennas and designing beamforming schemes to enhance the reliability of covert communication while increasing the uncertainty of detection at the source. The authors in [25] delved into a two-way relay covert communication scenario, where the system autonomously switches to the most suitable transmission strategy based on prior knowledge. Two-hop covert communication with an IRS-assisted relay was investigated in [26]–[28], enhancing covert performance through variables such as the number of IRS reflecting elements, the reflection coefficients, and the phase shift matrix at the IRS. Works [29], [30] studied selecting the optimal relay in scenarios with multiple relays to enhance covert capacity. Additionally, work [31] systematically analyzed how relays could choose between two transmission modes, half-duplex/full-duplex, and two forwarding modes, amplify-and-forward (AF)/decode-and-forward (DF), for optimal covert performance.

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Although the studies mentioned above have significantly contributed to the theoretical research of relay systems in covert communication, they all rely on an ideal assumption: the transmitter effortlessly possesses perfect channel state information (CSI) of covert channels. Our previous research has demonstrated that outdated CSI significantly impacts the covert performance in single-relay systems [34]. However, in multi-relay systems, the intricate issue of relay selection arises, leading to greater CSI delays than in single-relay systems. The effect of outdated CSI on covert performance in multi-relay systems remains unclear. Therefore, this paper is dedicated to exploring this issue. We aim to extend our understanding of how outdated CSI influences the effectiveness of covert communication strategies, particularly focusing on the complexities introduced in scenarios involving multiple relays. This exploration is essential for enhancing the robustness and efficiency of covert communication in increasingly complex network environments. Our main contributions can be summarized as follows:

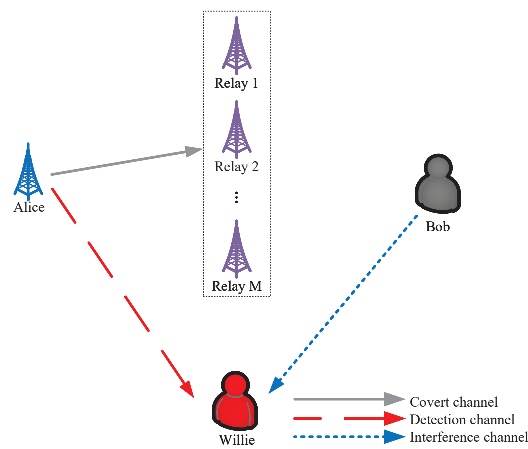
- By considering the scenario of CSI, we design a signaling procedure for multi-relay covert systems. This signaling procedure, developed under the premise of simplicity, minimizes the extent of CSI latency, thereby enhancing the system's covert performance. The design strategically addresses the challenges posed by outdated CSI, incorporating advanced algorithms to effectively update and utilize the available information.
- Utilizing the proposed signaling procedure, we develope theoretical models for Detection Error Probability (DEP) and Covert Rate (CR) under a power control scheme. These models intricately illustrate the fundamental relationship between outdated CSI and key communication performance metrics. This advancement not only provides a deeper understanding of the dynamics of outdated CSI in multi-relay systems but also offers a comprehensive framework for evaluating and optimizing covert communication strategies.
- We address an optimization problem concerning the trade-off between DEP and CR under outdated CSI. Utilizing a numerical search method, we provide a solution that offers insights for designing efficient and covert multi-relay systems. This approach enhances communication performance, balancing covertness and efficiency in environments with outdated CSI.

The rest of this article is organized as follows. Section II introduces the system model and preliminaries. Section III conducts the performance analysis and proposes the non-convex optimization problem. The numerical results are provided in Section IV. Finally, in Section V, we conclude this work.

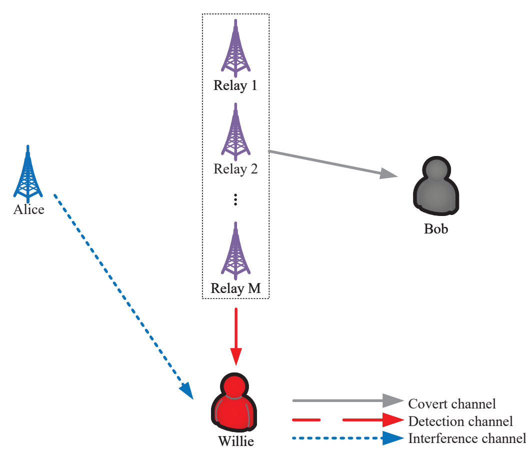
II. SYSTEM MODEL AND PRELIMINARIES

In our proposed covert communication system, illustrated in Fig. 1, we consider a setup consisting of Alice, Bob, Willie, and M decode-and-forward (DF) relays. Alice's objective is to covertly transmit a message to Bob with the assistance of one of the relays. Meanwhile, Willie's goal is to detect whether or not a message is being sent by Alice. Given that all nodes operate in half-duplex mode, each message transmission requires two hops to complete. To impede Willie's detection efforts, Bob transmits an interference signal to Willie during the first hop, and Alice does the same during the second hop.

The channel gain coefficient for the link $i \to j$ is denoted as h_{ij} , where $i, j \in \{a, r_m, b, w\}$ represent any two of the four nodes (Alice, Relay m, Bob and Willie). Note that all channels are subject to independent quasistatic Rayleigh block fading, so the channel fading coefficient follows an independent and identically distributed (i.i.d.) complex Gaussian distribution with zero mean and unit variance. To meet the actual communication scenario, we assume that Alice only knows h_{ar_m} and $h_{r_m b}$, but does not know h_{aw}.



(a) Phase 1 in each block



(b) Phase 2 in each block

Fig. 1. System model

A. Outdated CSI model

Notice that due to the time-varying channel property and processing delay in the training and feedback periods, Alice and Relays can only obtain outdated CSIs. We use h_{ij} to denote the instantaneous channel gain coefficient of $i \to j$ link, which is unknown to all nodes; we use $h_{ij}^{t-\mathcal{D}}$ to denote the estimated channel gain at the training period, which is just the outdated CSI used in our power control scheme

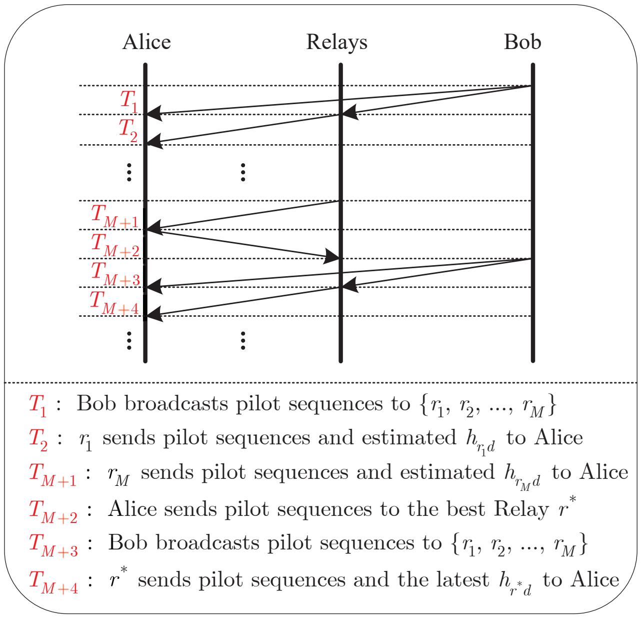


Fig. 2. Signaling procedure of relay selection.

for transmission scheduling. Based on the typical channel feedback delay model in [35]–[37], we can formulate $h^t_{ar_m}$ as

$$h^t_{ar_m} = \rho_{ar_m} h^{t-\mathcal{D}}_{ar_m} + \sqrt{1-\rho^2_{ar_m}}\,\omega^t_{ar_m} \tag{1}$$

and

$$h^t_{r_m b} = \rho_{r_m b} h^{t-\mathcal{D}}_{r_m b} + \sqrt{1-\rho^2_{r_m b}}\,\omega^t_{r_m b} \tag{2}$$

where $\mathcal{D}$ is the time difference between the channel estimation time and the current time, ω_{ij} is a circularly symmetric complex Gaussian random variable (RV) with the same variance as RV h_{ij} and is independent of RV h_{ij}, ρ_{ij} is the CC between h_{ij} and $h^{t-\mathcal{D}}_{ij}$. By using the Jakes' autocorrelation model [38], ρ_{ij} is given by

$$\rho_{ar_m} = J_0(2\pi f_{ar_m}\mathcal{D}) \tag{3}$$

and

$$\rho_{r_m b} = J_0(2\pi f_{r_m b}\mathcal{D}) \tag{4}$$

where $J_0()$ is the zeroth-order Bessel function of the first kind [39], f_{ij} is the maximum Doppler frequencies on the link $i \rightarrow j$,

B. Signaling procedure design

In a multi-relay system, a central node is required to possess CSIs of all links to select the most suitable relay for transmission in the current block. As depicted in Fig. 2, we assume Alice acts as this central node.

At the first time slot of each block, Bob broadcasts a pilot signal, enabling each relay to estimate its link CSI with Bob. In the subsequent M time slots, each of the M relays sequentially transmits a pilot signal and its previously estimated CSI to Alice. Concurrently, Alice estimates the link CSI with each relay. Without considering the latency of CSIs, the signaling process could conclude in $M+1$ time slots.

However, due to the channel's time-varying nature, the CSIs obtained by Alice are outdated, and their degree of latency varies because of the different feedback times from each relay. Utilizing severely outdated CSI for covert communication might lead to transmission interruption. To mitigate this issue, we introduce three additional time slots. In the $M+2$ time slot, Alice selects the optimal relay based on the outdated CSI and sends a pilot signal to it. In the $M+3$ time slot, Bob broadcasts another pilot signal, allowing every relay, including the optimal one, to estimate new CSI. Finally, in the $M+4$ timeslot, the optimal relay feeds back the latest two-hop link CSI to Alice.

C. Transmission Process

The transmission from a to b is implemented through the $a \rightarrow r$ transmission and $r \rightarrow b$ transmission. We now model the *imperfect signal-to-interference-noise ratio (SINR)* of $a \rightarrow r$ and $r \rightarrow b$, respectively.

$a \rightarrow r_m$ transmission: a transmits a vector of n symbols $\mathbf{x_a} = \{x^t_a\}^n_{t=1}$ with data transmit power P_a, where x^t_a is the t-th transmitted symbol satisfying $E[|x^t_a|^2] = 1$; the m-th relay receives a vector of n symbols $\mathbf{y_{r_m}} = \{y^t_r\}^n_{t=1}$, where t-th received symbol $y^t_{r_m}$ is given as

$$\begin{aligned} y^t_{r_m} &= \sqrt{P_a} h^t_{ar_m} x^t_a + n^t_{r_m} \\ &= \sqrt{P_a}(\rho_{ar_m} h^{t-\mathcal{D}}_{ar_m} + \sqrt{1-\rho^2_{ar_m}}\,\omega^t_{ar_m}) x^t_a + n^t_{r_m} \end{aligned} \tag{5}$$

where $n^t_{r_m} \sim CN(0, \sigma^2_{r_m})$ represents the AWGN at r_m. The received *outdated SINR* $\tilde{\gamma}_{sr}$ of a symbol x^t_a in the block at the m-th relay can be expressed as

$$\tilde{\gamma}_{ar_m} = \frac{P_a \rho^2_{ar_m} |h^{t-D}_{ar_m}|^2}{P_a(1-\rho^2_{ar_m})|\omega^t_{ar_m}|^2 + \sigma^2_{r_m}} \tag{6}$$

$r_m \rightarrow b$ transmission: The m-th relay forwards a vector of n symbols $\mathbf{x_{r_m}} = \{x^t_{r_m}\}^n_{t=1}$ with data transmit power P_{r_m}, where $x^t_{r_m}$ is the t-th transmitted symbol satisfying $E[|x^t_r|^2] = 1$; b receives a vector of n symbols $\mathbf{y_b} = \{y^t_b\}^{L_d}_{t=1}$, where the t-th received symbol y^t_b is given as

$$\begin{aligned} y^t_d &= \sqrt{P_{r_m}} h^t_{r_m d} x^t_{r_m} + n^t_d \\ &= \sqrt{P_{r_m}}(\rho_{r_m d} h^{t-\mathcal{D}}_{r_m d} + \sqrt{1-\rho^2_{r_m d}}\,\omega^t_{r_m d}) x^t_{r_m} + n^t_d \end{aligned} \tag{7}$$

where $n^t_b \sim CN(0, \sigma^2_b)$ represents the AWGN at b. The received *outdated SINR* $\tilde{\gamma}_{r_m b}$ of a symbol $x^t_{r_m}$ in the block at the m-th relay can be expressed as

$$\tilde{\gamma}_{r_m b} = \frac{P_{r_m} \rho^2_{r_m b} |h^{t-D}_{r_m b}|^2}{P_{r_m}(1-\rho^2_{r_m b})|\omega^t_{r_m b}|^2 + \sigma^2_b} \tag{8}$$

In the following discussion, we omit the time index $t-\mathcal{D}$ from $h^{t-\mathcal{D}}_{ij}$ for the sake of simplicity. The potential optimal relay can be selected according to the following rules

$$|h_m|^2 = \min\{|h_{ar_m}|^2, |h_{r_m d}|^2\}. \tag{9}$$

$$m^* = \underset{m}{\operatorname{argmax}}\{|h_m|^2\} \tag{10}$$

By considering the random transmit power scheme [40], the jamming power for Alice or Bob follows a uniform distribution

($P_j \sim U(0, P_j^{max})$), and the probability density function (PDF) is given by

$$f_{P_j}(x) = \begin{cases} \frac{1}{P_j^{max}}, & 0 < x < P_j^{max}, \\ 0, & otherwise, \end{cases} \tag{11}$$

D. Detection Strategy

Given that H_0 (the null hypothesis) and H_1 (the alternative hypothesis) are true, the received signals at Willie are respectively

$$H_0 : y_w^{(i)} = \sqrt{P_j} h_{\bar{k}w}^{(i)} x_j^{(i)} + n_w^{(i)}, \tag{12}$$

$$H_1 : y_w^{(i)} = \sqrt{P_a} h_{kw} x_k^{(i)} + \sqrt{P_j} h_{\bar{k}w}^{(i)} x_j^{(i)} + n_w^{(i)}, \tag{13}$$

where jamming-transmitting pair $(\bar{k}, k) \in \{(b, a), (a, r_m)\}$.

The optimal decision rule for minimizing the DEP at Willie can be determined by

$$T(n) = \frac{1}{n} \sum_{i=1}^{n} |y_w^{(i)}|^2 \underset{D_0}{\overset{D_1}{\gtrless}} \tau \tag{14}$$

where $T(n)$ is the received power in the use of n channels, $\hat{y}_s^{(i)}$ is the received signal in the ith channel use, τ is the detection threshold, D_1 and D_0 are the binary decisions that infer whether r transmits covert message or not, respectively. Considering Then, the DEP is given as

$$\xi \triangleq \mathcal{P}_{FA} + \mathcal{P}_{MD} \tag{15}$$

where $\mathcal{P}_{FA} = \mathbb{P}(D_1|H_0)$ and $\mathcal{P}_{MD} = \mathbb{P}(D_0|H_1)$ denote the false alarm probability and miss detection probability of Willie, respectively.

In this work, we consider the worst-case scenario where Willie adopts an optimal τ to minimize ξ. As such, the covert constraint considered in this work is $\xi^* \geq 1 - \epsilon$, where ξ^* is the minimum DEP at Willie and ϵ is the covertness requirement.

III. PERFORMANCE ANALYSIS

We first derive the ene-to-end successful transmission probability in multiple-relay system, then derive the DEP at Willie. Finally, we analyze the optimal CR under the covert constraint.

A. Successful Transmission Probability

Due to the randomness of transmit power at Alice and time-varying property of CSI, the outage event may occur, i.e., the channel capacity is less than predetermined tansmit rate. Instantaneous channel capacity of $a \to r_m$ and $r_m \to b$ are respectively expressed as

$$C_{ar_m} = \log_2(1 + \tilde{\gamma}_{ar_m}) \tag{16}$$

$$C_{r_m b} = \log_2(1 + \tilde{\gamma}_{r_m b}) \tag{17}$$

Conveniently, let $\tilde{\gamma}_{ar}^*$ denote the SNR from Alice to the best relay m^*; let $\tilde{\gamma}_{rb}^*$ denote the SNR from the best relay m^* to Bob. And we denote the corresponding channel capacity as C_{ar}^* and C_{rb}^*, respectively. In the DF relay systems with the power control scheme, the end-to-end successful transmission probability is determined as

$$\mathcal{P}_{su} = \mathbb{P}(C_{ar}^* \geq 2R_{ab}) \mathbb{P}(C_{rb}^* \geq 2R_{ab}), \tag{18}$$

By integrating (16) and (17) into (18), we drive the expression of $\mathcal{P}_{su}$ in (19), as shown at the top of the next page. Where functions of $|\omega_{ar_m}^t|^2$ and $|\omega_{r_m b}^t|^2$ are given by, respectively,

$$\phi_1(|\omega_{ar_m}^t|^2) = \frac{(4^{R_{ab}} - 1)\big(P_a(1 - \rho_{ar_m}^2)|\omega_{ar_m}^t|^2 + \sigma_{r_m}^2\big)}{P_a \rho_{ar_m}^2}, \tag{20}$$

$$\phi_2(|\omega_{r_m b}^t|^2) = \frac{(4^{R_{ab}} - 1)\big(P_{r_m}(1 - \rho_{r_m b}^2)|\omega_{r_m b}^t|^2 + \sigma_{r_m b}^2\big)}{P_a \rho_{r_m b}^2} \tag{21}$$

B. Detection Error Probability

Considering infinite channel use, by integrating (12) and (13) into (14),we have

$$\lim_{n \to \infty} T(n) = \begin{cases} P_j |h_{kw}|^2 + \sigma_w^2, & H_0 \\ P_k |h_{kw}|^2 + P_j |h_{\bar{k}w}|^2 + \sigma_w^2, & H_1 \end{cases} \tag{22}$$

By comparing the received signals with the given threshold τ, we can obtain $\mathcal{P}_{FA}$ and $\mathcal{P}_{MD}$ as following lemma.

***Lemma** 1:* For a given τ at willie, $\mathcal{P}_{FA}$ and $\mathcal{P}_{MD}$ can be given by

$$\mathcal{P}_{FA} = \begin{cases} 1, & \sigma_w^2 > \tau \\ 1 - \frac{\tau - \sigma_w^2}{P_j^{max} |h_{kw}|^2}, & \mu_1 \geq \tau \geq \sigma_w^2 \\ 0, & \tau > \mu_1 \end{cases} \tag{23}$$

$$\mathcal{P}_{MD} = \begin{cases} 1, & \tau > \mu_2 \\ \frac{\tau - \sigma_w^2 - P_k |h_{kw}|^2}{P_j^{max} |h_{\bar{k}w}|^2}, & \mu_2 \geq \tau \geq \sigma_w^2 \\ 0, & \sigma_w^2 > \tau \end{cases} \tag{24}$$

where $\mu_1 \triangleq P_j^{max} |h_{kw}|^2 + \sigma_w^2$, $\mu_2 \triangleq P_k |h_{kw}|^2 + P_j^{max} |h_{\bar{k}w}|^2 + \sigma_w^2$.

Proof 3.1: According to the received power $T(n)$ in (22), the false alarm probability $\mathcal{P}_{FA}$ is calculated as

$$\begin{aligned} \mathcal{P}_{FA} &= \mathbb{P}(D_1|H_0) = \mathbb{P}\Big[P_j |h_{kw}|^2 + \sigma_w^2 > \tau\Big] \\ &= \begin{cases} 1, & \sigma_w^2 > \tau \\ 1 - \mathbb{P}\Big[P_j < \frac{\tau - \sigma_w^2}{|h_{kw}|^2}\Big], & \mu_1 \geq \tau \geq \sigma_w^2 \\ 0, & \tau > \mu_1 \end{cases} \end{aligned} \tag{25}$$

Substituting (11) into (25), (23) is achieved. Similarly, the miss detection probability $\mathcal{P}_{MD}$ is calculated as

$$\begin{aligned} \mathcal{P}_{MD} &= \mathbb{P}(D_0|H_1) \\ &= \mathbb{P}\Big[P_k |h_{kw}|^2 + P_j |h_{\bar{k}w}|^2 + \sigma_w^2 < \tau\Big] \\ &= \begin{cases} 1, & \tau > \mu_2 \\ \mathbb{P}\Big[P_j < \frac{\tau - \sigma_w^2 - P_k |h_{kw}|^2}{|h_{\bar{k}w}|^2}\Big], & \mu_2 \geq \tau \geq \sigma_w^2 \\ 0, & \sigma_w^2 > \tau \end{cases} \end{aligned} \tag{26}$$

Substituting (11) into (26), we can obtain (24).

$$
\begin{aligned}
\mathcal{P}_{su} &= \mathbb{P}(\tilde{\gamma}^*_{ar} > 4^{R_{ab}} - 1)\mathbb{P}(\tilde{\gamma}^*_{rb} > 4^{R_{ab}} - 1) \\
&= 1 - \prod_{m=1}^{M} \mathbb{P}(\tilde{\gamma}_{ar_m} < 4^{R_{ab}} - 1)\mathbb{P}(\tilde{\gamma}^*_{rb} < 4^{R_{ab}} - 1) \\
&= 1 - \prod_{m=1}^{M} \int_0^{\infty} e^{-|\omega^t_{ar_m}|^2} \int_0^{\phi_1(|\omega^t_{ar_m}|^2)} e^{-|h_{ar_m}|^2} d|h_{ar_m}|^2 d|\omega^t_{ar_m}|^2 \times \int_0^{\infty} e^{-|\omega^t_{r_m d}|^2} \int_0^{\phi_2(|\omega^t_{r_m d}|^2)} e^{-|h_{r_m d}|^2} d|h_{r_m d}|^2 d|\omega^t_{r_m d}|^2 \\
&= 1 - \prod_{m=1}^{M} \frac{(1-\rho^2_{ar_m})\exp\left\{\frac{\sigma^2_r(4^{R_{ab}}-1)-\rho^2_{ar}}{(4^{R_{ab}}-1)P_a(1-\rho^2_{ar_m})}\right\} - \exp\left\{\frac{\sigma^2_{r_m}(4^{R_{ab}}-1)-\rho^2_{ar_m}}{(4^{R_{ab}}-1)P_a}\right\}}{1-\rho^2_{ar_m}} \\
&\times \frac{(1-\rho^2_{r_m b})\exp\left\{\frac{\sigma^2_r(4^{R_{ab}}-1)-\rho^2_{r_m b}}{(4^{R_{ab}}-1)P_{r_m}(1-\rho^2_{r_m b})}\right\} - \exp\left\{\frac{\sigma^2_{r_m}(4^{R_{ab}}-1)-\rho^2_{r_m b}}{(4^{R_{ab}}-1)P_{r_m}}\right\}}{1-\rho^2_{r_m b}}
\end{aligned} \tag{19}
$$

Substituting (23) and (24) into (15), Willie's DEP can be given by

$$
\xi = \begin{cases} 1 + \frac{(\tau+\sigma^2_w)(|h_{kw}|^2-|h_{\bar{k}w}|^2)-P_k|h_{kw}|^4}{P^{max}_j|h_{\bar{k}w}|^2|h_{kw}|^2}, & \mu_1 \geq \tau > \sigma^2_w \\ \frac{\tau-\sigma^2_w-P_k|h_{kw}|^2}{P^{max}_j|h_{\bar{k}w}|^2}, & \mu_2 > \tau > \mu_1 \\ 1, & otherwise, \end{cases} \tag{27}
$$

where μ_1 and μ_2 are given in Lemma 1. According to (27), we can know that the global optimal threshold may exist in the interval $(\sigma^2_w, \mu_1]$ or $(\mu_2, \mu_1]$, such that we need to discuss each piece separately. When $\tau \in (\mu_1, \mu_2]$, the first-order derivative of ξ with respect to τ can be given as

$$
\frac{\partial \xi}{\partial \tau} = \frac{1}{P^{max}_j|h_{\bar{k}w}|^2} > 0 \tag{28}
$$

In this interval, $\partial\xi/\partial\tau$ is always greater than 0, meaning that the DEP increases with the rise of the detection threshold. Therefore, the optimal detection threshold must be within the range that is less than μ_1. Similarly, when $\tau \in (\sigma^2_w, \mu_1]$, we get the first-order derivative of ξ with respect to τ as

$$
\frac{\partial \xi}{\partial \tau} = \frac{|h_{kw}|^2 - |h_{\bar{k}w}|^2}{P^{max}_j|h_{\bar{k}w}|^2|h_{kw}|^2} \tag{29}
$$

In this context, the sign of $\partial\xi/\partial\tau$ is indeterminate and is determined by the CSI from Willie to the transmitter and from Willie to the jammer at each moment. So we use the numerical search to find the local optimal in this interval, which also is the global optimal threshold for the minimum of ξ. Thus the optimal detection threshold can be determined as

$$
\tau^* \triangleq \underset{\mu_1 \geq \tau > \sigma^2_w}{\operatorname{argmin}} \; 1 + \frac{(\tau+\sigma^2_w)(|h_{kw}|^2-|h_{\bar{k}w}|^2)-P_k|h_{kw}|^4}{P^{max}_j|h_{\bar{k}w}|^2|h_{kw}|^2}. \tag{30}
$$

C. Optimal Problem of Maximal CR and RR

Based on the probability $\mathcal{P}_{su}$ and transmission rate R_{ab} from Alice to Bob for covert messages transmission, covert rate (CR) $\bar{R}_{ab}$ can be determined as

$$
\bar{R}_{ab} = \mathcal{P}_{su} R_{ab}. \tag{31}
$$

TABLE I
SIMULATION PARAMETER SETTING

Parameter	Value
Noise level σ^2_j (dBm)	-10
Alice → Relay m maximum transmit power P^{max}_a(dBm)	20
Relay m → Bob maximum transmit power $P^{max}_{r_m}$(dBm)	20
Alice → Relay m minimum transmit power P^{min}_a(dBm)	1
Relay m → Bob maximum transmit power $P^{min}_{r_m}$(dBm)	1
Alice → Bob predetermined rate R_{ab} (bits per channel use)	1
Covertness requirement ϵ	0.1

The covert performance metric in terms of CR and DEP, which are functions of both Alice's transmit power P_a, and the m-th relay's transmit power P_{r_m}, then the optimization problem can be formulated as

$$
\underset{P_a, P_{r_m}}{\text{maximize}} \; \bar{R}_{ab}(P_a, P_{r_m}) \tag{32a}
$$

$$
s.t. \quad P^{max}_a \geq P_a \geq P^{min}_a \tag{32b}
$$

$$
P^{max}_{r_m} \geq P_{r_m} \geq P^{min}_{r_m} \tag{32c}
$$

$$
\xi^*(P_a, P_{r_m}) \geq 1 - \epsilon, \tag{32d}
$$

Constraints (32b) and (32c) show that according to the data transmission, the power of both nodes has an upper bound. Constraint (32d) is used to ensure that the minimum DEP is greater than some value under the optimal detection threshold, ε is predetermined to specify the covertness constraint. The optimization problem in (32) can be solved by numerical search to obtain the maximum CR denoted as $\bar{R}^*_c$. Formally, we summarize the numerical search algorithm in Algorithm 1.

IV. NUMERICAL RESULTS

In this section, we conduct simulations to verify the efficiency of the theoretical performance analysis, and then provide comprehensive numerical results to illustrate the DEP performance and CR performance under various extent of CSI latency. Unless otherwise specified, the parameter configuration is set as Table I.

As shown in Fig. 3, we analyzed how the minimum Detection Error Probability (DEP) varies with the number of relays. It is observed that, under various degrees of CSI latency, the

Algorithm 1 Maximum CR Algorithm

Input: Given transmission rate R_{ab}; Accuracy parameter of Alice's transmit power Δ_1; Accuracy parameter of the m-th relay's transmit power Δ_2; The maximal of jamming power P_j^{max} and other environmental variables.

Output: Optimal Alice's transmit power P_a, the m-th relay's transmit power P_{r_m}.

```
Compute P_su according to (19);
for l = i; P_{r_m} < P_{r_m}^{max}; k++ do
    P_{r_m} = P_{r_m} + kΔ_1
    for k = i; P_a < P_a^{max}; k++ do
        P_a = P_a + kΔ_2
        Update ξ*(P_a, P_{r_m}, P_j^{max}) according to (27);
        if ξ*(P_a, P_{r_m}, P_j^{max}) < 1 − ϵ then
            R̄_ab(k, l) = 0; Continue;
        end if
        Update R̄_ab(k, l) according to (31);
    end for
    P_a^* = argmax_k R̄_ab(k);
end for
P_{r_m}^* = argmax_l R̄_ab(l);
```

DEP increases as the number of relays grows, but the rate of increase gradually slows down. This indicates that augmenting the number of relays can effectively enhance the detection error probability at Willie, thereby improving covertness. However, the potential for this enhancement reaches saturation after a certain number of relays is added. Furthermore, we notice that the curve for $\rho_{ij} = 0.9$ consistently lies above that for $\rho_{ij} = 0.6$, implying that lower latency degrees correspond to stronger covert performance.

Additionally, we analyzed how the optimal Covert Rate (CR) varies with the number of relays. As shown in Fig. 4, it is observed that under different degrees of CSI latency, the optimal CR also increases as the number of relays is augmented, but the rate of this increase slows down over time. This suggests that increasing the number of relays can effectively enhance the covert rate. However, the room for such enhancement reaches a saturation point as the number of relays is increased to a certain extent. Moreover, we note that the slope of the curve for $\rho_{ij} = 0.9$ is greater than that for $\rho_{ij} = 0.6$, indicating that lower levels of latency lead to more effective improvements in covert rate through the addition of more relays.

V. CONCLUSIONS

In this study, we have successfully addressed critical challenges in multi-relay covert communication systems under the constraint of outdated CSI. Our key contribution is the development of a simplified signaling procedure that effectively minimizes the impact of CSI latency, substantially enhancing covert performance. Through this novel procedure, we have constructed theoretical models to elucidate the relationship between outdated CSI and vital communication metrics like Detection Error Probability (DEP) and Covert Rate (CR). These models provide profound insights into the dynamics of outdated CSI, offering a robust framework for evaluating and optimizing covert communication strategies.

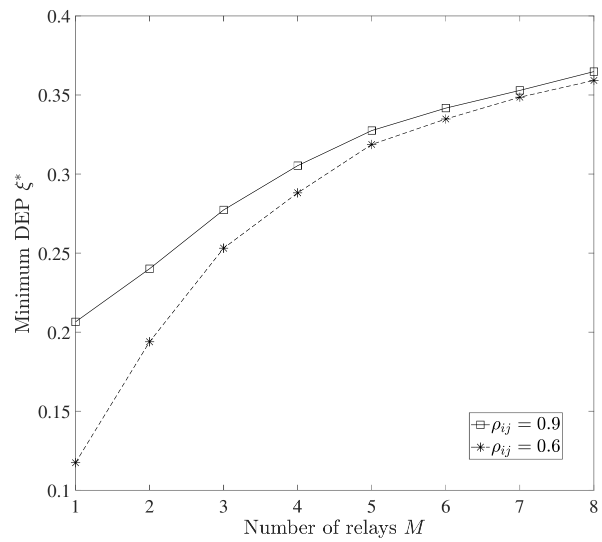


Fig. 3. Minimum DEP ξ^* vs. Number of relays M.

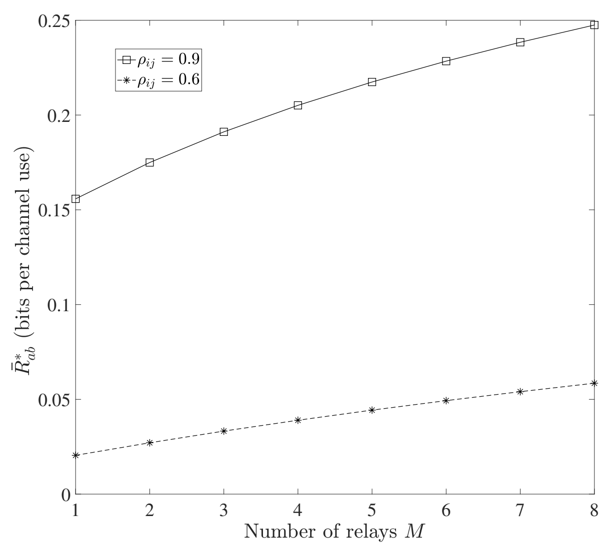


Fig. 4. CR $\bar{R}_{ab}^*$ vs. Number of relays M.

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