IEC
Institute of Electronics
and Computer

Journal of the Institute of Electronics and Computer, 2021, 3, 87–97
https://iecscience.org/journals/JIEC
ISSN Online: 2643–8240

A Robust Weighted Distance Measure and its Applications in Decision-making via Pythagorean Fuzzy Information

Paul Augustine Ejegwa[1,*], **Idoko Charles Onyeke**[2]

[1]Department of Mathematics, University of Agriculture, P.M.B. 2373, Makurdi, Nigeria
Email: ejegwa.augustine@uam.edu.ng
[2]Department of Computer Science, University of Agriculture, P.M.B. 2373, Makurdi, Nigeria
Email: onyeke.idoko@uam.edu.ng
*Corresponding Author: P.A. Ejegwa, Email: ejegwa.augustine@uam.edu.ng

How to cite this paper: Paul Augustine Ejegwa and Idoko Charles Onyeke (2021) A Robust Weighted Distance Measure and its Applications in Decision-making via Pythagorean Fuzzy Information. Journal of the Institute of Electronics and Computer, 3, 87–97.
https://doi.org/10.33969/JIEC.2021.31007

Received: September 20, 2021
Accepted: October 15, 2021
Published: October 18, 2021

Abstract

Pythagorean fuzzy set (PFS) has proven to be a competent soft computing tool because of its capacity to tackle fuzziness in decision-making. Pythagorean fuzzy distance measures are reliable techniques deployed to appreciate the application of PFSs. Some distance measures between PFSs have been explored, where the complete parameters of PFSs are considered. These distance measures lack reliability due to the negligent of the weights of elements under Pythagorean fuzzy situation. In this paper, a novel distance measure between PFSs is proposed and its weighted version to enhance reliability in terms of applications. To show the suitability of the measures, we characterize the distance measure and its weighted version with some results. In addition, certain decision-making problems involving cases of pattern recognition and disease diagnosis are discussed based on the measures. From a comparative analysis of some existing distance measures with the novel distance measures, it is observed that the proposed distance measures are superior in term of accuracy and reliability.

Keywords

Decision-making, Distance measure, Pythagorean fuzzy set, Pattern recognition, Disease diagnosis

1. Introduction

Decision-making is a herculean task enmeshed with fuzziness. The introduction of fuzzy set (FS) [1] enhanced the solution of many decision-making problems. FS though significant has a drawback in the sense that it considers only the membership degree μ (MD) of the case under consideration. Because of this drawback, Atanassov [2] proposed a generalized fuzzy set known as intuitionistic fuzzy set (IFS). IFS is described by membership degree μ, nonmembership degree ν and intuitionistic fuzzy index π with the property that their sum is one. IFSs have been applied in sundry cases [3–8]. The concept of distance measures under intuitionistic fuzzy context have been discussed as reliable information measures [9–12]. Some existing distance measures were revised and applied to medical diagnostic process [13], and sundry applications of distance measures between IFSs have been studied [14, 15].

Albeit, the situation where the sum of MD and NMD is more than one is beyond the scope of IFS. For instance, if $\mu = \frac{\sqrt{3}}{2}$ and $\nu = \frac{1}{2}$, then IFS is handicapped to model such a problem. The shortcoming in IFSs naturally led to the introduction of intuitionistic fuzzy set of the second type (IFSST) [16], which is referred to Pythagorean fuzzy sets (PFSs) [17, 18]. PFS provides a new approach to deal with vagueness considering MD μ and NMD ν satisfying the conditions; $\mu + \nu \geq 1$ and $\mu^2 + \nu^2 \leq 1$. PFS has near relationship with IFS. Because of the flexibility of the notion of PFSs, it has been used to address some real-life problems [19–23].

Distance measure is a soft computing technique use to find the distance between two arbitrary PFSs akin to metric function. Distance measures have been utilized in resolving many real-life problems in Pythagorean fuzzy domain. Zhang and Xu [21] initiated the study of distance measure in Pythagorean fuzzy context by proposing a distance measure and applied it to multiple criteria decision making. Li and Zeng [24] introduced a new distance measure between PFSs with real-life applications. Some distance measures between PFSs have been introduced and characterized [25]. The method of calculating distance between PFSs in [21] was modified in [26] for better output. Several other distance measures between PFSs have been studied and applied to multiple criteria group decision-making [27, 28].

The distance measures between IFSs/PFSs studied in [11, 21, 25, 26] are very appropriate because they captured the three parameters of IFSs/PFSs to avoid information loss. Albeit, these distance measures lack reliability due to the negligent of the weights of elements, which can negatively affect the outputs. Thus, the motivation of this study is to introduce weighted distance measure between PFSs with better performance index compare to the existing distance measures [11, 21, 25, 26]. The specific objectives of this work includes; (i) explore some existing distance measures in Pythagorean fuzzy domain, (ii) propose new distance measure and its weighted version between PFSs, (iii) apply the proposed distances in cases involving pattern recognition and disease diagnosis, (iv) present comparison of the new distances for PFSs with the existing distance measures. The paper is thus outlined; Section 2 presents some mathematical background of PFSs and discusses some existing distances in Pythagorean fuzzy setting, Section 3 introduces the new distances between PFSs and their properties, Section 4 discusses the applications of the proposed distances in cases involving pattern recognition and disease diagnosis, and Section 5 draws conclusion with recommendation for further studies.

2. Preliminaries

This section presents some mathematical background of PFSs and discusses some distance measures for PFSs.

2.1. Pythagorean fuzzy sets

Myriad of works have been done on the mathematical background of IFSs and PFSs. Here, some basic concepts of PFSs are presented to be used in the subsequent sections. Let us assume that X is a non-empty set throughout this paper.

Definition 2.1. [29] An intuitionistic fuzzy set $\mathbb{A}$ of X is defined by

$$\mathbb{A} = \{\langle x, \mu_{\mathbb{A}}(x), \nu_{\mathbb{A}}(x)\rangle : x \in X\}, \tag{1}$$

where $\mu_{\mathbb{A}},\ \nu_{\mathbb{A}} : X \to [0,1]$ are MD and NMD of $x \in X$, and $0 \leq \mu_{\mathbb{A}}(x) + \nu_{\mathbb{A}}(x) \leq 1$. For an IFS $\mathbb{A}$ in X, $\pi_{\mathbb{A}}(x) \in [0,1] = 1 - \mu_{\mathbb{A}}(x) - \nu_{\mathbb{A}}(x)$ is the intuitionistic fuzzy index or hesitation margin of $\mathbb{A}$.

Definition 2.2. [18] A Pythagorean fuzzy set $\mathbb{B}$ of X is defined by

$$\mathbb{B} = \{\langle x, \mu_{\mathbb{B}}(x), \nu_{\mathbb{B}}(x)\rangle : x \in X\}, \tag{2}$$

where $\mu_{\mathbb{B}},\ \nu_{\mathbb{B}}: X \to [0,1]$ are MD and NMD of $x \in X$, and $0 \leq \mu^2_{\mathbb{B}}(x) + \nu^2_{\mathbb{B}}(x) \leq 1$. For a PFS $\mathbb{B}$ in X, $\pi_{\mathbb{B}}(x) \in [0,1] = \sqrt{1 - \mu^2_{\mathbb{B}}(x) - \nu^2_{\mathbb{B}}(x)}$ is the Pythagorean fuzzy index or hesitation margin of $\mathbb{B}$.

Definition 2.3. [18] Suppose $\mathbb{B}$ and $\mathbb{C}$ are PFSs in X, then for all $x \in X$ we have
(i) $\mathbb{B} = \mathbb{C}$ iff $\mu_{\mathbb{B}}(x) = \mu_{\mathbb{C}}(x)$, $\nu_{\mathbb{B}}(x) = \nu_{\mathbb{C}}(x)$.
(ii) $\mathbb{B} \subseteq \mathbb{C}$ iff $\mu_{\mathbb{B}}(x) \leq \mu\mathbb{C}(x)$, $\nu_{\mathbb{B}}(x) \geq \nu_{\mathbb{C}}(x)$.
(iii) $\overline{B} = \{\langle x, \nu_{\mathbb{B}}(x), \mu_{\mathbb{B}}(x)\rangle : x \in X\}$.
(iv) $\mathbb{B} \cup \mathbb{C} = \{\langle x, \max(\mu_{\mathbb{B}}(x), \mu_{\mathbb{C}}(x)), \min(\nu_{\mathbb{B}}(x), \nu_{\mathbb{C}}(x))\rangle : x \in X\}$.
(v) $\mathbb{B} \cap \mathbb{C} = \{\langle x, \min(\mu_{\mathbb{B}}(x), \mu_{\mathbb{C}}(x)), \max(\nu_{\mathbb{B}}(x), \nu_{\mathbb{C}}(x))\rangle : x \in X\}$.

Definition 2.4. [19] Pythagorean fuzzy pair (PFP) is characterized by the form $\langle b, c\rangle$ such that $b + c \leq 1$ where $b, c \in [0,1]$. PFP evaluate the PFS for which the components (b and c) are interpreted as MD and NMD.

2.2. Distances between Pythagorean fuzzy sets

Distance measure is a soft computing technique use in the applications of PFSs. The definition of distance measure between PFSs is given thus.

Definition 2.5. [25] If $\mathbb{B}$ and $\mathbb{C}$ are PFSs of X, then the distance between $\mathbb{B}$ and $\mathbb{C}$ denoted by $d(\mathbb{B},\mathbb{C})$ is a function $d: PFS \times PFS \to [0,1]$ which satisfies
(i) $0 \leq d(\mathbb{B},\mathbb{C}) \leq 1$
(ii) $d(\mathbb{B},\mathbb{C}) = 0$ iff $\mathbb{B} = \mathbb{C}$
(iii) $d(\mathbb{B},\mathbb{C}) = d(\mathbb{C},\mathbb{B})$
(iv) $d(\mathbb{B},\mathbb{D}) \leq d(\mathbb{B},\mathbb{C}) + d(\mathbb{C},\mathbb{D})$, where $\mathbb{D}$ is also a PFS of X.

When $d(\mathbb{B},\mathbb{C})$ reaches 0, it shows that $\mathbb{B}$ and $\mathbb{C}$ are more close or related. Again, if $d(\mathbb{B},\mathbb{C})$ reaches 1 then $\mathbb{B}$ and $\mathbb{C}$ are not related or close. For any two PFSs $\mathbb{B}$ and $\mathbb{C}$ of $X = \{x_1, \cdots, x_n\}$, we present the following distances between them.

2.2.1. Burillo and Bustince distances

By extending the distances in [30], Burillo and Bustince [9] proposed the following distances:

$$d_1(\mathbb{B},\mathbb{C}) = \frac{1}{2}\Sigma_{i=1}^n \Big(|\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i)| + |\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i)|\Big) \tag{3}$$

$$d_2(\mathbb{B},\mathbb{C}) = \Big(\frac{1}{2}\Sigma_{i=1}^n \big((\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 + (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2\big)\Big)^{0.5} \tag{4}$$

$$d_3(\mathbb{B},\mathbb{C}) = \frac{1}{2n}\Sigma_{i=1}^n \Big(|\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i)| + |\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i)|\Big) \tag{5}$$

$$d_4(\mathbb{B},\mathbb{C}) = \Big(\frac{1}{2n}\Sigma_{i=1}^n \big((\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 + (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2\big)\Big)^{0.5} \tag{6}$$

The obvious limitation of these approaches [9] is that the hesitation margin is not considered in the computations.

2.2.2. Szmidt and Kacprzyk distances

The modifications of the approaches in [9] were presented in [11] by incorporating hesitation margin, namely:

$$d_5(\mathbb{B},\mathbb{C}) = \frac{1}{2}\Sigma_{i=1}^{n}\Big(|\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i)| + |\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i)| + |\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i)|\Big) \quad (7)$$

$$d_6(\mathbb{B},\mathbb{C}) = \Big(\frac{1}{2}\Sigma_{i=1}^{n}\big((\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 + (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2 + (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i))^2\big)\Big)^{0.5} \quad (8)$$

$$d_7(\mathbb{B},\mathbb{C}) = \frac{1}{2n}\Sigma_{i=1}^{n}\Big(|\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i)| + |\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i)| + |\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i)|\Big) \quad (9)$$

$$d_8(\mathbb{B},\mathbb{C}) = \Big(\frac{1}{2n}\Sigma_{i=1}^{n}\big((\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 + (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2 + (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i))^2\big)\Big)^{0.5} \quad (10)$$

2.2.3. Zhang and Xu distance

In [21], a distance measure between PFSs was proposed, i.e.,

$$d_9(\mathbb{B},\mathbb{C}) = \frac{1}{2}\Sigma_{i=1}^{n}\Big(|\mu^2_{\mathbb{B}}(x_i) - \mu^2_{\mathbb{C}}(x_i)| + |\nu^2_{\mathbb{B}}(x_i) - \nu^2_{\mathbb{C}}(x_i)| + |\pi^2_{\mathbb{B}}(x_i) - \pi^2_{\mathbb{C}}(x_i)|\Big). \quad (11)$$

2.2.4. Modified Zhang and Xu distance

In [26], a distance measure between PFSs was proposed which normalized the distance measure in [21]. The distance is given by

$$d_{10}(\mathbb{B},\mathbb{C}) = \frac{1}{2n}\Sigma_{i=1}^{n}\Big(|\mu^2_{\mathbb{B}}(x_i) - \mu^2_{\mathbb{C}}(x_i)| + |\nu^2_{\mathbb{B}}(x_i) - \nu^2_{\mathbb{C}}(x_i)| + |\pi^2_{\mathbb{B}}(x_i) - \pi^2_{\mathbb{C}}(x_i)|\Big). \quad (12)$$

3. Weighted distance measure between Pythagorean fuzzy sets

Now, we present a new distance measure between PFSs and its weighted version to enhance reliability. For $X = \{x_1, \cdots, x_n\}$, the new distance measure between two PFSs $\mathbb{B}$ and $\mathbb{C}$ of X is

$$\mathbf{d}(\mathbb{B},\mathbb{C}) = \Big(\frac{1}{3n}\Sigma_{i=1}^{n}\big[(\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 + (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2 + (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i))^2\big]\Big)^{0.5} \quad (13)$$

Eq. (13) captures the complete parameters of PFSs and also takes cognizance of the number of parameters as seen in the denominator.

To avoid unreliable output, the weights of the elements $x_i \in X$ for $i = 1, \cdots, n$ should be considered. Assume the weights of $x_i \in X$ is α_i for $i = 1, \cdots, n$ where $0 \leq \alpha_i \leq 1$ and $\sum_{i=1}^{n} \alpha_i = 1$. Thus the weighted version of Eq. (13) is as follows:

$$\mathbf{d}_{\alpha}(\mathbb{B},\mathbb{C}) = \Big(\frac{1}{3n}\Sigma_{i=1}^{n}\alpha_i\big[(\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 + (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2 + (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i))^2\big]\Big)^{0.5} \quad (14)$$

Theorem 3.1. *If $\mathbb{B}$ and $\mathbb{C}$ are PFSs in X. Then*

(i) $\boldsymbol{d}_\alpha(\mathbb{B},\mathbb{C}) = \boldsymbol{d}_\alpha(\mathbb{C},\mathbb{B})$,
(ii) $\boldsymbol{d}_\alpha(\mathbb{B},\mathbb{C}) = \boldsymbol{d}_\alpha(\overline{\mathbb{B}},\overline{\mathbb{C}})$.

Proof. The proofs of (i) and (ii) are straightforward.

Theorem 3.2. *Let $\mathbb{B}$ and $\mathbb{C}$ be PFSs in X. Then $\boldsymbol{d}_\alpha(\mathbb{B},\mathbb{C})$ satisfies the conditions of distance measure between $\mathbb{B}$ and $\mathbb{C}$.*

Proof. The proof of (i) of Definition 2.5 is straightforward. Now, we prove (ii) of Definition 2.5. Suppose $\mathbf{d}_\alpha(\mathbb{B},\mathbb{C}) = 0$. Then

$$(\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 = 0,\ (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2 = 0,\ \text{and}\ (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i))^2 = 0.$$

Thus

$$\mu_{\mathbb{B}}(x_i) = \mu_{\mathbb{C}}(x_i),\ \nu_{\mathbb{B}}(x_i) = \nu_{\mathbb{C}}(x_i)\ \text{and}\ \pi_{\mathbb{B}}(x_i) = \pi_{\mathbb{C}}(x_i),$$

and hence $\mathbb{B} = \mathbb{C}$. The converse is straightforward. Thus (ii) of Definition 2.5 follows. Again, since

$$\begin{aligned}\mathbf{d}_\alpha(\mathbb{B},\mathbb{C}) &= \Big(\frac{\sum_{i=1}^n \alpha_i[(\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 + (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2 + (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i))^2]}{3n}\Big)^{0.5} \\ &= \Big(\frac{\sum_{i=1}^n \alpha_i[(\mu_{\mathbb{C}}(x_i) - \mu_{\mathbb{B}}(x_i))^2 + (\nu_{\mathbb{C}}(x_i) - \nu_{\mathbb{B}}(x_i))^2 + (\pi_{\mathbb{C}}(x_i) - \pi_{\mathbb{B}}(x_i))^2]}{3n}\Big)^{0.5},\end{aligned}$$

it implies that $\mathbf{d}_\alpha(\mathbb{B},\mathbb{C}) = \mathbf{d}_\alpha(\mathbb{C},\mathbb{B})$, and hence (iii) of Definition 2.5 as desired.

Suppose $\mathbb{D}$ is also a PFS of X, then the distances $\mathbf{d}_\alpha(\mathbb{B},\mathbb{C})$, $\mathbf{d}_\alpha(\mathbb{B},\mathbb{D})$ and $\mathbf{d}_\alpha(\mathbb{C},\mathbb{D})$ satisfy the triangle inequality if $\mathbf{d}_\alpha(\mathbb{B},\mathbb{D}) \leq \mathbf{d}_\alpha(\mathbb{B},\mathbb{C}) + \mathbf{d}_\alpha(\mathbb{C},\mathbb{D})$. To see this, if

$$\begin{aligned}\mathbf{d}_\alpha(\mathbb{B},\mathbb{D}) &= \max\Big(\frac{1}{3n}\sum_{i=1}^n [(\mu_{\mathbb{B}}(x_k) - \mu_{\mathbb{D}}(x_k))^2 + (\nu_{\mathbb{B}}(x_k) - \nu_{\mathbb{D}}(x_k))^2 + (\pi_{\mathbb{B}}(x_k) - \pi_{\mathbb{D}}(x_k))^2]\Big)^{0.5} \\ &= \Big(\frac{1}{3n}\sum_{i=1}^n [(\mu_{\mathbb{B}}(x_k) - \mu_{\mathbb{D}}(x_k))^2 + (\nu_{\mathbb{B}}(x_k) - \nu_{\mathbb{D}}(x_k))^2 + (\pi_{\mathbb{B}}(x_k) - \pi_{\mathbb{D}}(x_k))^2]\Big)^{0.5}\end{aligned}$$

for some fixed k, $1 \leq k \leq n$ i.e., the maximum is attained at k. Then

$$\begin{aligned}(\mu_{\mathbb{B}}(x_k) - \mu_{\mathbb{D}}(x_k))^2 &\leq (\mu_{\mathbb{B}}(x_k) - \mu_{\mathbb{C}}(x_k))^2 + (\mu_{\mathbb{C}}(x_k) - \mu_{\mathbb{D}}(x_k))^2, \\ (\nu_{\mathbb{B}}(x_k) - \nu_{\mathbb{D}}(x_k))^2 &\leq (\nu_{\mathbb{B}}(x_k) - \nu_{\mathbb{C}}(x_k))^2 + (\nu_{\mathbb{C}}(x_k) - \nu_{\mathbb{D}}(x_k))^2, \\ (\pi_{\mathbb{B}}(x_k) - \pi_{\mathbb{D}}(x_k))^2 &\leq (\pi_{\mathbb{B}}(x_k) - \pi_{\mathbb{C}}(x_k))^2 + (\pi_{\mathbb{C}}(x_k) - \pi_{\mathbb{D}}(x_k))^2.\end{aligned}$$

Thus $\mathbf{d}_\alpha(\mathbb{B},\mathbb{D}) \leq \mathbf{d}_\alpha(\mathbb{B},\mathbb{C}) + \mathbf{d}_\alpha(\mathbb{C},\mathbb{D})$ as desired. Hence the properties of distance measure are satisfied.

Theorem 3.3. *Suppose $\mathbb{B}$, $\mathbb{C}$ and $\mathbb{D}$ are PFSs in X with the properties $\mathbb{B} \subseteq \mathbb{C} \subseteq \mathbb{D}$. Then*
(i) $\boldsymbol{d}_\alpha(\mathbb{B},\mathbb{D}) \geq \boldsymbol{d}_\alpha(\mathbb{B},\mathbb{C})$,
(ii) $\boldsymbol{d}_\alpha(\mathbb{B},\mathbb{D}) \geq \boldsymbol{d}_\alpha(\mathbb{C},\mathbb{D})$,
(iii) $\boldsymbol{d}_\alpha(\mathbb{B},\mathbb{D}) \geq \max[\boldsymbol{d}_\alpha(\mathbb{B},\mathbb{C}), \boldsymbol{d}_\alpha(\mathbb{C},\mathbb{D})]$.

Proof. Because $\mathbb{B} \subseteq \mathbb{C} \subseteq \mathbb{D}$, we have

$$\begin{aligned}(\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{D}}(x_i))^2 &\geq (\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2, \\ (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{D}}(x_i))^2 &\geq (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2, \\ (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{D}}(x_i))^2 &\geq (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i))^2.\end{aligned}$$

Thus

$$\begin{aligned}(\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{D}}(x_i))^2 &+ (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{D}}(x_i))^2 + (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{D}}(x_i))^2 \\ &\geq (\mu_{\mathbb{B}}(x_i) - \mu_{\mathbb{C}}(x_i))^2 + (\nu_{\mathbb{B}}(x_i) - \nu_{\mathbb{C}}(x_i))^2 \\ &+ (\pi_{\mathbb{B}}(x_i) - \pi_{\mathbb{C}}(x_i))^2.\end{aligned}$$

So, $\mathbf{d}_\alpha(\mathbb{B},\mathbb{D}) \geq \mathbf{d}_\alpha(\mathbb{B},\mathbb{C})$, and so (i) holds. With the same argument, $\mathbf{d}_\alpha(\mathbb{B},\mathbb{D}) \geq \mathbf{d}_\alpha(\mathbb{C},\mathbb{D})$, which proves (ii). From (i) and (ii), $\mathbf{d}_\alpha(\mathbb{B},\mathbb{D}) \geq \max[\mathbf{d}_\alpha(\mathbb{B},\mathbb{C}), \mathbf{d}_\alpha(\mathbb{C},\mathbb{D})]$, i.e., (iii) follows.

4. Applicative Illustrations

This section addresses applications of the new distance measures and the existing distance measures under PFSs in decision-making problems of pattern recognition and disease diagnosis. Suppose there are m choices represented in PFPs C_j for $j = 1,\ldots,m$ considered in a feature space S. If there is a sample choice denoted as PFP D to be associated with C_j, then the value of

$$\mathbf{d}(C_j,D) = \min\left[\mathbf{d}(C_1,D),\cdots,\mathbf{d}(C_m,D)\right] \tag{15}$$

or

$$\mathbf{d}_\alpha(C_j,D) = \min\left[\mathbf{d}_\alpha(C_1,D),\cdots,\mathbf{d}_\alpha(C_m,D)\right] \tag{16}$$

where $\mathbf{d}(C_j,D)$ (or $\mathbf{d}_\alpha(C_j,D)$) indicates the classification of C_j and D.

4.1. Case of pattern recognition

The process of identifying patterns via machine learning procedure is incorporated with uncertainties. Thus the approach of pattern recognition based on Pythagorean fuzzy information is an interesting technique for reliable pattern classification. Suppose there are three patterns P_1, P_2 and P_3, represented as PFPs in $S = \{s_1,s_2,s_3\}$ with weights $\alpha = \{0.3,0.4,0.3\}$. If there is an unidentified pattern Q represented in PFP in the same feature space S. The representations of the patterns are in Table 1.

Table 1. Pattern representations

PFPs	Feature space		
	s_1	s_2	s_3
μ_{P_1}	0.1000	0.5000	0.1000
ν_{P_1}	0.1000	0.1000	0.9000
π_{P_1}	0.9899	0.8602	0.4243
μ_{P_2}	0.5000	0.7000	0.0000
ν_{P_2}	0.5000	0.3000	0.8000
π_{P_2}	0.7071	0.6481	0.6000
μ_{P_3}	0.7000	0.1000	0.4000
ν_{P_3}	0.2000	0.8000	0.4000
π_{P_3}	0.6856	0.5916	0.8246
μ_Q	0.4000	0.6000	0.0000
ν_Q	0.4000	0.2000	0.8000
π_Q	0.8246	0.7746	0.6000

Then our task is to classify Q into any of P_j, $j = 1,2,3$, by deploying the existing distance measures and the new distance measures.

Using Szmidt and Kacprzyk distance [11]: we get

$$d_5(P_1,Q) = 0.7133,\ d_5(P_2,Q) = 0.3220,\ d_5(P_3,Q) = 1.4733 \text{ using Eq. (7)}$$

$$d_6(P_1,Q) = 0.3778,\ d_6(P_2,Q) = 0.1868,\ d_6(P_3,Q) = 0.7626 \text{ using Eq. (8)}$$

$$d_7(P_1,Q) = 0.2378,\ d_7(P_2,Q) = 0.1073,\ d_7(P_3,Q) = 0.4911 \text{ using Eq. (9)}$$

$$d_8(P_1,Q) = 0.2181,\ d_8(P_2,Q) = 0.1079,\ d_8(P_3,Q) = 0.4403 \text{ using Eq. (10).}$$

By using Zhang and Xu distance [21], we get $d_9(P_1,Q) = 0.6199$, $d_9(P_2,Q) = 0.3600$, $d_9(P_3,Q) = 1.4099$. By using modified Zhang and Xu distance [26], we get $d_{10}(P_1,Q) =$

0.2066, $d_{10}(P_2,Q) = 0.1200$, $d_{10}(P_3,Q) = 0.4700$. By using the new distance measure, we get $\mathbf{d}(P_1,Q) = 0.1781$, $\mathbf{d}(P_2,Q) = 0.0881$, $\mathbf{d}(P_3,Q) = 0.3595$. Using the weighted distance measure, we get $\mathbf{d}_\alpha(P_1,Q) = 0.0991$, $\mathbf{d}_\alpha(P_2,Q) = 0.0522$, $\mathbf{d}_\alpha(P_3,Q) = 0.2143$.

From the computations, Eqs. (7) and (8) of Szmidt and Kacprzyk distance are weak distance measures, whereas the new distance measure and its weighted version are the most reliable distance measures.

The results of the distances between the known patterns and the unidentified pattern are presented in Table 2.

Table 2. Results of distance measures

Methods	Pattern classifications		
	(P_1,Q)	(P_2,Q)	(P_3,Q)
Szmidt and Kacprzyk [11]	0.7133	0.3220	1.4733
	0.3778	0.1868	0.7626
	0.2378	0.1073	0.4911
	0.2181	0.1079	0.4403
Zhang and Xu [21]	0.6199	0.3600	1.4099
Ejegwa [26]	0.2066	0.1200	0.4700
New method	**0.1781**	**0.0881**	**0.3595**
New weighted method	**0.0991**	**0.0522**	**0.2143**

From Table 2, the unidentified pattern Q belongs to pattern P_1 since $d(P_3,Q) > d(P_1,Q) > d(P_2,Q)$ for all the distance methods.

4.2. Case of disease diagnosis

Deploying Pythagorean fuzzy decision-making approach to disease diagnosis is necessary because of the uncertainties involve in the process. Since disease diagnosis is a critical assignment, care should be taken to avoid wrong diagnosis. Thus, we present a disease diagnosis based on distance measures using Pythagorean fuzzy medical information.

Suppose we have a set of diseases

$$D = \{\text{viral fever, malaria, typhoid fever, stomach pain, chest pain}\}$$

represented in PFPs, and a set of symptoms $S = \{s_1, s_2, s_3, s_4, s_5\}$ where s_1 = temperature, s_2 = headache, s_3 = stomach pain, s_4 = cough, s_5 = chest pain, which are the clinical expressions of D. Taking the weights of the symptoms of D to be $\alpha = \{0.1, 0.15, 0.3, 0.2, 0.25\}$.

Assume a patient P expresses some symptoms in S and his/her Pythagorean fuzzy medical information is known. Table 3 contains Pythagorean fuzzy information of D_j, $j = 1, \ldots, 5$ and P with respect to S.

Table 3. Pythagorean fuzzy medical information

PFPs	Clinical expressions				
	s_1	s_2	s_3	s_4	s_5
μ_V	0.4000	0.3000	0.1000	0.4000	0.1000
ν_V	0.0000	0.5000	0.7000	0.3000	0.7000
π_V	0.9165	0.8124	0.7071	0.8660	0.7071
μ_M	0.7000	0.2000	0.0000	0.7000	0.1000
ν_M	0.0000	0.6000	0.9000	0.0000	0.8000
π_M	0.7141	0.7746	0.4342	0.7141	0.5916
μ_T	0.3000	0.6000	0.2000	0.2000	0.1000
ν_T	0.3000	0.1000	0.7000	0.6000	0.9000
π_T	0.9055	0.7937	0.6856	0.7746	0.4243
μ_S	0.1000	0.2000	0.8000	0.2000	0.2000
ν_S	0.7000	0.4000	0.0000	0.7000	0.7000
π_S	0.7071	0.8944	0.6000	0.6856	0.6856
μ_C	0.1000	0.0000	0.2000	0.2000	0.8000
ν_C	0.8000	0.8000	0.8000	0.8000	0.1000
π_C	0.5916	0.6000	0.5657	0.5657	0.5916
μ_P	0.6000	0.5000	0.3000	0.7000	0.3000
ν_P	0.1000	0.4000	0.4000	0.2000	0.4000
π_P	0.7937	0.7681	0.8660	0.6856	0.8660

Note: V is for viral fever, M is for malaria, T is for typhoid fever, S is for stomach pain and C is for chest pain, respectively. Now, we diagnose the disease of P by finding which of the diseases has the shortest distance to P by deploying the existing distance measures and the new distance measures. After computations, the results are presented in Table 4.

Table 4. Results of distance measures

Methods	Pattern classifications				
	(V,P)	(M,P)	(T,P)	(S,P)	(C,P)
Szmidt and Kacprzyk [11]	1.3327	1.5596	1.8743	2.1797	2.7824
	0.5292	0.7062	0.7996	0.9583	1.0020
	0.2665	0.3119	0.3749	0.4359	0.5565
	0.2367	0.3158	0.3576	0.4286	0.4481
Zhang and Xu [21]	1.3599	1.5099	1.8499	2.0199	2.7399
Ejegwa [26]	0.2720	0.3020	0.3700	0.4040	0.5480
New method	**0.1932**	**0.2579**	**0.2920**	**0.3499**	**0.3659**
New weighted method	**0.0917**	**0.0919**	**0.1183**	**0.1538**	**0.1733**

From Table 4, one can suggests that the patient P is suffering from viral fever. The results show amazing relationship between viral fever, malaria and typhoid fever. Thus, the patient should also be treated for malaria and typhoid fever because the patient is significantly closed to the diseases.

4.3. Comparative analysis

The results in Tables 2 and 4 shows that the Eqs. (7) and (8) in [11] and the method in [21] are not reliable distance measures between PFSs since they violate condition (i) of Definition 2.5. The proposed methods show high reliability indexes compare to the methods in [11, 21, 26]. Most especially, the weighted distance measure is the most reliable measure because of its high accuracy due to the considerations of weights. This agrees to the significant of weights of elements in the computation of distances between PFSs.

5. Conclusion

This paper has shown the capacity of PFSs in tackling uncertainties in decision-making problems based on distance measure approaches. We proposed a new distance measure between PFSs and its weighted version to enhance reliability in application situations. Applications of the studied distance measures are demonstrated in cases of pattern recognition and disease diagnosis. By comparing the novel distances with the existing distance measures in terms of the applications, the proposed approaches yield reliable results with better performance indexes. These new distances could be studied in interval-valued Pythagorean fuzzy sets, picture fuzzy sets, q-rung orthopair fuzzy sets, etc. for future endeavour.

Acknowledgements

The contributions of the anonymous reviewers are highly appreciated.

Conflicts of Interest

This work has no conflict of interest whatsoever.

References

[1] Zadeh, L.A. (1965) Fuzzy Sets. *Information and Control*, 8, 338–353.

[2] Atanassov, K.T. (1986) Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems*, 20, 87–96.

[3] Atanassov, K.T. (1999) Intuitionistic Fuzzy Sets: Theory and Applications. Physica-Verlag, Heidelberg.

[4] Ejegwa, P.A. and Onasanya, B.O. (2019) Improved intuitionistic fuzzy composite relation and its application to medical diagnostic process. *Notes on Intuitionistic Fuzzy Sets*, 25(1), 43–58.

[5] Ejegwa, P.A. (2021) Novel Correlation Coefficient for Intuitionistic Fuzzy Sets and its Application to Multi-criteria Decision-making Problems. *International Journal of Fuzzy System Applications*, 10(2), 39–58.

[6] Ejegwa, P.A., Onoja, A.M. and Chukwukelu, S.N. (2014) Application of Intuitionistic Fuzzy Sets in Research Questionnaire. *Journal of Global Research in Mathematical Archives*, 2(5), 51–54.

[7] Liu, P. and Chen, S.M. (2017) Group Decision Making based on Heronian Aggregation Operators of Intuitionistic Fuzzy Numbers. *IEEE Transaction on Cybernetics*, 47(9), 2514–2530.

[8] Szmidt, E. and Kacprzyk, J. (2001) Intuitionistic Fuzzy Sets in Some Medical Applications. *Notes on Intuitionistic Fuzzy Sets*, 7(4), 58–64.

[9] Burillo, P. and Bustince, H. (1996) Entropy on Intuitionistic Fuzzy Sets and on Interval-valued Fuzzy Sets. *Fuzzy Sets and Systems*, 78, 305–315.

[10] Hatzimichailidis, A.G., Papakostas, A.G. and Kaburlasos, V.G. (2012) A Novel Distance Measure of Intuitionistic Fuzzy Sets and its Application to Pattern Recognition Problems. *International Journal of Intelligent Systems*, 27, 396–409.

[11] Szmidt, E. and Kacprzyk, J. (2000) Distances between Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems*, 114, 505–518.

[12] Wang, W. and Xi, X. (2005) Distance Measure between Intuitionistic Fuzzy Sets. *Pattern Recognition Letters*, 26, 2063–2069.

[13] Davvaz, B. and Sadrabadi, E.H. (2016) An Application of Intuitionistic Fuzzy Sets in Medicine. *International Journal of Biomathematics*, 9(3), 1650037.

[14] Ejegwa, P.A., Chukwukelu, S.N. and Odoh, D.E. (2014) Test of Accuracy of Some Distance Measures use in the Application of Intuitionistic Fuzzy Sets in Medical Diagnosis. *Journal of Global Research in Mathematical Archives*, 2(5), 55–60.

[15] Ejegwa, P.A. and Onyeke, I.C. (2018) An Object Oriented Approach to the Application of Intuitionistic Fuzzy Sets in Competency based Test Evaluation. *Annals of Communications in Mathematics*, 1(1), 38–47.

[16] Atanassov, K.T. (1989) Geometrical Interpretation of the Elements of the Intuitionistic Fuzzy Objects. Preprint IM-MFAIS-1-89, Sofia.

[17] Yager, R.R. (2013) Pythagorean Membership Grades in Multicriteria Decision Making. Technical Report MII-3301 Machine Intelligence Institute, Iona College, New Rochelle, NY.

[18] Yager, R.R. and Abbasov, A.M. (2013) Pythagorean Membership Grades, Complex Numbers and Decision Making. *International Journal of Intelligent and Systems*, 28(5) 436–452.

[19] Ejegwa, P.A. and Jana, C. (2021) Some New Weighted Correlation Coefficients between Pythagorean Fuzzy Sets and their Applications, In: Garg, H. (Eds.); Pythagorean fuzzy sets, pp. 39–64, Springer.

[20] Ejegwa, P.A., Adah, V. and Onyeke, I.C. (2021) Some Modified Pythagorean Fuzzy Correlation Measures with Application in Determining Some Selected Decision-making Problems. *Granular Computing*, DOI: 10.1007/s41066-021-00272-4.

[21] Zhang, X.L., and Xu, Z.S. (2014) Extension of TOPSIS to Multiple Criteria Decision Making with Pythagorean Fuzzy Sets. *International Journal Intelligent and Systems*, 29, 1061–1078.

[22] Liang, D., and Xu, Z. (2017) The New Extension of TOPSIS Method for Multiple Criteria Decision Making with Hesitant Pythagorean Fuzzy Sets. Applied and Soft Computing, 60, 167–179.

[23] Garg, H. (2016) A New Generalized Pythagorean Fuzzy Information Aggregation using Einstein Operations and its Application to Decision Making. *International Journal Intelligent and Systems*, 31(9), 886–920.

[24] Li, D.Q. and Zeng, W.Y. (2018) Distance Measure of Pythagorean Fuzzy Sets. *International Journal Intelligent and Systems*, 33, 348–361.

[25] Ejegwa, P.A. (2020) Distance and Similarity Measures for Pythagorean Fuzzy Sets. *Granular Computing*, 5(2), 225–238.

[26] Ejegwa, P.A. (2020) Modified Zhang and Xu's Distance Measure of Pythagorean Fuzzy Sets and its Application to Pattern Recognition Problems. *Neural Computing and Applications*, 32(14), 10199–10208.

[27] Zeng, W., Li, D. and Yin, Q. (2018) Distance and Similarity Measures of Pythagorean Fuzzy Sets and their Applications to Multiple Criteria Group Decision Making. *International Journal Intelligent and Systems*, DOI: 10.1002 /int. 22027.

[28] Ejegwa, P.A. and Awolola, J.A. (2021) Novel Distance Measures for Pythagorean Fuzzy Sets with Applications to Pattern Recognition Problems. *Granular Computing*, 6, 181–189.

[29] Atanassov, K.T. (1994) New Operations Defined on Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems*, 61, 137–142.

[30] Diamond, P. and Kloeden, P. (1994) Metric spaces of fuzzy sets theory and applications. Singapore: Word Scientific.