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An Improved Fuzzy Inventory Model Under Two Warehouses

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Abstract

The objective of this work is to present an improved inventory system with fuzzy constraints dealing with two warehouses system-own and rented. In the present model, we analyze the system under the consideration of two warehouses and without shortages with the assumptions of the linear demand function (increasing function of time). Generally, in today's business scenario for sessional products, some constraints like storage cost, deteriorating cost, and ordering cost change with their original values. Therefore, these constraints cannot be assumed to be constant in that situation. Depending on these facts that we handle these costs as a triangular fuzzy number and hence apply the signed distance technique to solve the corresponding problem. The key objective of this work is to determine the optimal inventory level, and inventory time schedule to a minimum of the whole inventory cost. The proposed model is demonstrated with two numerical examples to observe the behavior of constraints with system cost and compare their performance with and without fuzzy environment.

Keywords

Inventory, Two warehouses, fuzzy sets, linear demand, fuzzy total cost.

1. Introduction

Inventory theory is one of the most important aspect for any decision-making process which involves many parameters such as demand cost, shortage cost, deterioration rate and so on. Among all these parameters, a demand plays a key role in any inventory management process. In the literature, many researchers have considered different kinds of models with different demands rates while constructing the inventory model. However, as the systems of the various industries are complex in these days and hence their products and output demands are not fixed at all. In other words, in the realistic conditions, the formulation of the new products or items and their quality, prices and costs are not always fixed in nature due to the deficiency of the exact form of the information. To handle such ambiguity in the formulation, a fuzzy set theory [26] plays a key role. All the parameters associated with the inventory models are considered as a fuzzy parameter to describe their uncertainties in a well manner. In previous years the fuzzy theory has been successfully implemented in various engineering, medical and business problems. The fuzzy theory has the potential to provide a better result in comparison to a crisp environment. For inventory management, the fuzzy theory is a ground-breaking solution method to transform the optimize results.

Uncertainty, in its mathematical logic, states to fluctuate the values of any parameters. The fuzzy theory has been practicing very successfully to an extensive range of irregular behavior of the parameters like costs in inventory management, industries, stock market, and business organization. Uncertainty research has usually attentive on discussing the irregular behavior of constraints which break down the supply and demand inventory system of any business organization for the product. Initially, the fuzzy theory discussed by Zadeh [26] and after that some improvement with applications in fuzzy theory [8]. An unreliable production system inventory model under fuzzy demand rate using signed distance method [9]. The investigation of fuzzy values in the inventory work is to facilitate to the expert for determine the economic fuzzy quantities [25]. The fuzzy theory concepts is demonstrated by examples connecting decision making process for deterministic and stochastic scenario [1]. The fuzzy inventory model using K-T Conditions and graded mean integration representation method is demonstrated for economic order quantity in which trapezoidal fuzzy numbers is used for expressed the quantities and costs [2]. They presented a method for fuzzy inventory cost with demand rate is trapezoidal fuzzy number [3]. The advancement of inventory model has progressed with fuzzy constraints [11-15].

In the present business scenario, demand and deterioration are mainly in fuzzy numbers due to fluctuation with many factors. Especially for a sessional product, availability of the product and its price dependent supply. Many-times, retailer faces the various problem as a result loss in business due to increase in the cost of the product. Inventory model with the fuzzy environment using certain constraints such as storage cost, price and ordering cost [5-7]. For sessional products, the price fluctuation is continued due to increases and decreases in costs. For sessional products may be future production or sales, the concept of two warehouse with EOQ model discussed by [10]. We find researchers who worked to improve the comprehension of the tradition inventory model under two warehouses and its application in several fields with maintain the originality [19-21, 24]. In the literature, some authors have already work on inventory model for determining the optimality policies with variable constraints [18, 22, 23]. In this perspective, demonstrate an improved inventory model under fuzzy constraints and warehouse capacity [4, 16-17].

Under the consideration of the above work and the present work give an improved fuzzy inventory model to describe the behavior of the system two warehouses with linear demand. In real life, we all the time see that the customer satisfaction is the main factor for the present business scenario for any inventory system. Due to uncertainty of the costs, deterioration, prices, cannot be assumed to be fixed values. In those respects, we consider the demand rate of the products are linear in the model. Further, a triangular fuzzy number is considered to describe the uncertainties in the data and hence the optimization model is taken as fuzzy optimization model in which the objective of the work is to minimize the total inventory cost.

The rest of the work is summarized as follows: In section 2, we briefly present some assumptions and notations. In section 3, we describe the crisp model of the system while in section 4, we deal with the fuzzy model with the signed distance method. In section 5, we validate the applicability and relevant of the developed model for minimum the inventory cost in crisp and fuzzy environment with constraints involved and analyze thereafter our optimum result. Finally, in section 6, we will condense the research work and magnet some conclusion around the further research works.

2. Assumptions and Notations

The proposed improved fuzzy inventory model established with subsequent assumptions and notations {~ sign represent the fuzzy constraints}.

2.1. Assumptions

The following assumptions are considered during the formulation.

(i) The proposed inventory model considers only one product.

(ii) The demand of the product is $m_1 + m_2 t$.

(iii) The lead time is considered to be zero and shortages are not considered.

(iv) The proposed model considers two warehouses: own and rent warehouses.

(v) Consider two positive fuzzy numbers $\tilde{X}$ and $\tilde{Y}$; $\tilde{X} \oplus \tilde{Y} = (x_1 y_1, x_2 y_2, x_3 y_3, x_4 y_4)$, where $\tilde{X} = (x_1, x_2, x_3, x_4)$ and $\tilde{Y} = (y_1, y_2, y_3, y_4)$.

(vi) Consider a triangular fuzzy number $\lambda = (\lambda_1, \lambda_2, \lambda_3)$, where $\lambda_1 = \lambda - \Delta_1, \lambda_2 = \lambda, \lambda_3 = \lambda + \Delta_2$.

The membership function of λ is $\mu_{\lambda}\left(\tilde{\lambda}\right)=\begin{cases}\dfrac{\lambda-\lambda_1}{\lambda_2-\lambda_1} & \lambda_1\le\lambda\le\lambda_2\\ \dfrac{\lambda_3-\lambda}{\lambda_3-\lambda_2} & \lambda_2\le\lambda\le\lambda_3\\ 0 & \text{otherwise}\end{cases}.$

2.2. Notation

OW: Own warehouse
RW: Rent warehouse
d_o : ordering cost per order
x : deterioration rate in own warehouse
y : deterioration rate in rent warehouse; $x>y$.
d_c : deteriorating cost per unit in OW/RW
η_1 : inventory storage cost in RW
η_2 : inventory storage cost in OW; $\eta_1>\eta_2$.
W_o : maximum inventory level in OW
TIC : total inventory system cost.
$dTIC$: total fuzzy inventory cost using signed distance

3. Crisp Model

In the proposed model, we consider an inventory system under two warehouses system for sessional products. For RW, the inventory level N_r reach at zero level after the time t_1. During period (0, t_1), the demand of the customer fulfils from RW, so in between some product deteriorate in OW in same time period represents the inventory level N_{o1}. After rent warehouse empty, the customer demand fulfils by OW during time period (t_1, T=t_1+t_2) represents the inventory level N_{o2}. The initial inventory level for own warehouse is W_O. The framework of the proposed two warehouse inventory model is described in Figure 1.
Therefore, the governing equations of the system considering the boundary conditions

$N_r(t_1)=0,\ N_{o1}(0)=W_O,\ N_{o2}(T)=0$ is

$$\frac{dN_r}{dt}+xN_r=-\left(m_1+m_2t\right),\qquad 0\le t\le t_1 \qquad (1)$$

$$\frac{dN_{o1}}{dt}+yN_{o1}=0,\qquad 0\le t\le t_1 \qquad (2)$$

$$\frac{dN_{o2}}{dt}+yN_{o2}=-\left(m_1+m_2t\right), \qquad t_1\le t\le T \quad (3)$$

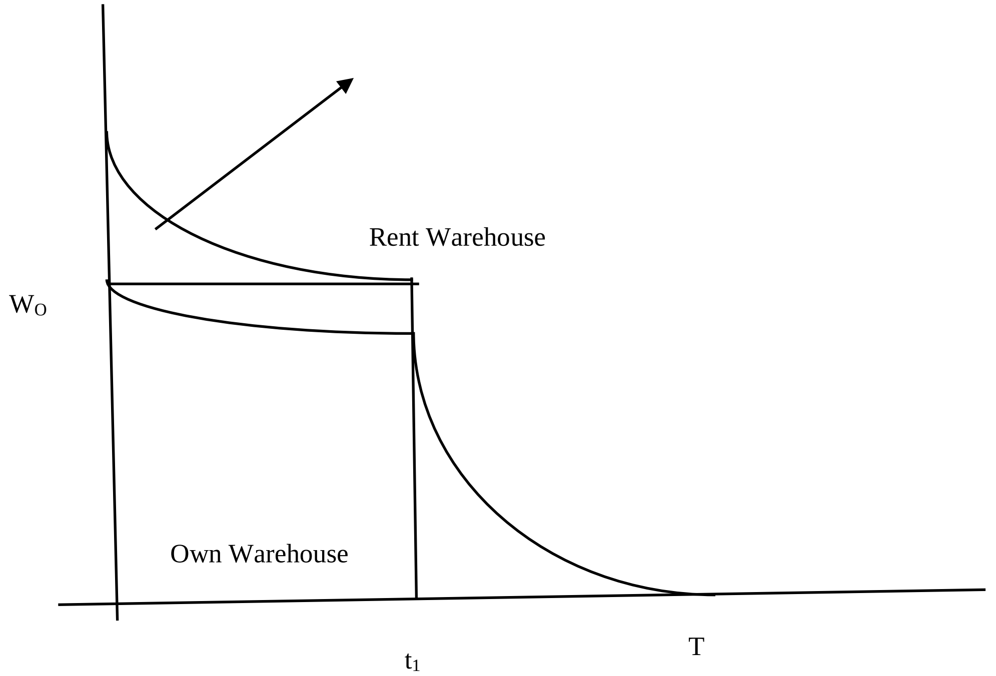

Figure 1. Proposed Two warehouses Inventory Model

After simplification, we can obtain the solution of the above equations as

$$N_r\left(t\right)=-\frac{m_1+m_2t}{x}+\frac{m_2}{x^2}+\left(\frac{m_1+m_2t}{x}-\frac{m_2}{x^2}\right)e^{x\left(t_1-t\right)} \qquad (4)$$

$$N_{o1}\left(t\right)=W_O\,e^{-yt} \quad (5)$$

$$N_{o2}\left(t\right)=-\frac{m_1+m_2t_1}{y}+\frac{m_2}{y^2}+\left(\frac{m_1+m_2T}{y}-\frac{m_2}{y^2}\right)e^{y\left(T-t\right)} \quad (6)$$

From the model, at $t=t_1;\, N_{o1}\left(t_1\right)=N_{o2}\left(t_1\right)$, thus we have

$$W_O=-\left(\frac{m_1+m_2t_1}{y}-\frac{m_2}{y^2}\right)e^{yt_1}+\left(\frac{m_1+m_2T}{y}-\frac{m_2}{y^2}\right)e^{yT} \quad (7)$$

Further, Inventory ordering cost (IOC) is described as

$$IOC=d_o \qquad (8)$$

and the Inventory storage cost (ISC) in RW is defined as

$$ISC_R=\eta_1\int_0^{t_1}N_r\left(t\right)dt$$

$$= \eta_1 \left[-\frac{m_1}{x^2}(1+xt_1) - \frac{m_2}{x}\left(\frac{t_1^{\,2}}{2} - \frac{1}{x^2}\right) + \left(\frac{m_1+m_2 t}{x^2} - \frac{m_2}{x^3}\right)e^{xt_1} \right] \quad (9)$$

Similarly, Inventory storage cost in OW is stated as

$$ISC_O = \eta_2 \left[\int_0^{t_1} N_{o1}(t)\,dt + \int_{t_1}^{T} N_{o2}(t)\,dt \right]$$

$$= \eta_2 \begin{bmatrix} \frac{W_O}{y}\left(1-e^{-yt_1}\right) - \frac{m_1}{y^2}(1+yt_2) \\ -\frac{m_2}{y}\left(\frac{t_2^{\,2}}{2} + t_1 t_2 + \frac{t_1}{y} - \frac{1}{y^2}\right) + \left(\frac{m_1}{y} + \frac{m_2 T}{y} - \frac{m_2}{y^2}\right)e^{yt_2} \end{bmatrix} \quad (10)$$

and the Inventory deterioration cost in RW is given as

$$IDC_R = d_c \int_0^{t_1} x.N_r(t)\,dt$$

$$= x\, d_c \left[-\frac{m_1}{x^2}(1+xt_1) - \frac{m_2}{x}\left(\frac{t_1^{\,2}}{2} - \frac{1}{x^2}\right) + \left(\frac{m_1+m_2 t}{x^2} - \frac{m_2}{x^3}\right)e^{xt_1} \right] (11)$$

Inventory deterioration cost in OW is

$$ISC_O = d_c \left[\int_0^{t_1} y.N_{o1}(t)\,dt + \int_{t_1}^{T} y.N_{o2}(t)\,dt \right]$$

$$= y\, d_c \begin{bmatrix} \frac{W_O}{y}\left(1-e^{-yt_1}\right) - \frac{m_1}{y^2}(1+yt_2) \\ -\frac{m_2}{y}\left(\frac{t_2^{\,2}}{2} + t_1 t_2 + \frac{t_1}{y} - \frac{1}{y^2}\right) + \left(\frac{m_1}{y} + \frac{m_2 T}{y} - \frac{m_2}{y^2}\right)e^{yt_2} \end{bmatrix} (12)$$

Therefore, the total inventory cost (TIC) is

$$TIC = \frac{1}{T}\left[\mathrm{IOC} + \mathrm{ISC_R} + \mathrm{ISC_R} + \mathrm{IDC_R} + \mathrm{IDC_O}\right]$$

$$= \frac{1}{T}\left\{ d_o + (\eta_1 + xd_c)\left[-\frac{m_1}{x^2}(1+xt_1) - \frac{m_2}{x}\left(\frac{t_1^{\,2}}{2} - \frac{1}{x^2}\right) + \left(\frac{m_1+m_2 t}{x^2} - \frac{m_2}{x^3}\right)e^{xt_1} \right] \right.$$

$$\left. + (\eta_2 + y\, d_c) \begin{bmatrix} \frac{W_O}{y}\left(1-e^{-yt_1}\right) - \frac{m_1}{y^2}(1+yt_2) \\ -\frac{m_2}{y}\left(\frac{t_2^{\,2}}{2} + t_1 t_2 + \frac{t_1}{y} - \frac{1}{y^2}\right) + \left(\frac{m_1}{y} + \frac{m_2 T}{y} - \frac{m_2}{y^2}\right)e^{yt_2} \end{bmatrix} \right\} (13)$$

For determining the optimal values of total inventory cost use the essential and sufficient conditions for minimum cost:

$$\frac{\partial TIC}{\partial t_1}=0,\ \frac{\partial TIC}{\partial t_2}=0,$$

$$\left(\frac{\partial^2 TIC_1}{\partial t_1{}^2}\right)\left(\frac{\partial^2 TIC_1}{\partial t_2{}^2}\right)-\left(\frac{\partial^2 TIC_1}{\partial t_1\partial t_2}\right)^2>0 \text{ and } \frac{\partial^2 TIC}{\partial t_1{}^2}>0.$$

4. Fuzzy inventory model

In this section, presented the fuzzy inventory model through the signed distance method. Due to sessional products and global market scenario the price of products fluctuates with their ordering, storage and deterioration cost that means these costs constraints cannot be assumed fix. To avoid this type of problem we consider these cost for the both warehouses-own and rent warehouses as a triangular fuzzy number. For developing this improved inventory model, assumed the following triangular fuzzy numbers for ordering cost, deterioration cost and storage cost:

1. $d_o \in [d_o-\Delta_1, d_o+\Delta_2]$, where $0<\Delta_1<d_o$ and $0<\Delta_1\Delta_2$.
2. $d_c \in [d_c-\Delta_3, d_c+\Delta_4]$, where $0<\Delta_3<d_c$ and $0<\Delta_3\Delta_4$.
3. $\eta_1 \in [\eta_1-\Delta_5, \eta_1+\Delta_6]$, where $0<\Delta_5<\eta_1$ and $0<\Delta_5\Delta_6$.
4. $\eta_2 \in [\eta_2-\Delta_7, \eta_2+\Delta_8]$, where $0<\Delta_7<\eta_2$ and $0<\Delta_7\Delta_8$.

Signed distance method for the proposed model is

1. $d(\tilde{d}_o, 0)=d_o+\frac{1}{4}(\Delta_2-\Delta_1)$ 2. $d(\tilde{d}_c, 0)=d_c+\frac{1}{4}(\Delta_4-\Delta_3)$

3. $d(\tilde{\eta}_1, 0)=\eta_1+\frac{1}{4}(\Delta_6-\Delta_5)$ 4. $d(\tilde{\eta}_2, 0)=\eta_2+\frac{1}{4}(\Delta_8-\Delta_7)$

Using such information, we can transform the crisp model of the total cost into the fuzzy environment. From equation (13), we can deduce that $T\tilde{I}C=(T\tilde{I}C_1, T\tilde{I}C_2, T\tilde{I}C_3)$, where

$$T\tilde{I}C_1=\frac{1}{T}\Bigg\{(d_o-\Delta_1)+\left[(\eta_1-\Delta_5)+x(d_c-\Delta_3)\right]\left[\begin{array}{l}-\frac{m_1}{x^2}(1+xt_1)-\frac{m_2}{x}\left(\frac{t_1^2}{2}-\frac{1}{x^2}\right)\\+\left(\frac{m_1+m_2t}{x^2}-\frac{m_2}{x^3}\right)e^{xt_1}\end{array}\right]+\left[(\eta_2-\Delta_7)+y(d_c-\Delta_3)\right]$$
$$\left[\begin{array}{l}\frac{W_O}{y}\left(1-e^{-yt_1}\right)-\frac{m_1}{y^2}(1+yt_2)\\-\frac{m_2}{y}\left(\frac{t_2^2}{2}+t_1t_2+\frac{t_1}{y}-\frac{1}{y^2}\right)+\left(\frac{m_1}{y}+\frac{m_2T}{y}-\frac{m_2}{y^2}\right)e^{yt_2}\end{array}\right]\Bigg\} \quad (14)$$

$$T\tilde{I}C_2=T\tilde{I}C \quad (15)$$

$$\tilde{TIC}_3 = \frac{1}{T}\Big\{\left(d_o+\Delta_2\right)+\left[\left(\eta_1+\Delta_6\right)+x\left(d_c+\Delta_4\right)\right]$$

$$\begin{bmatrix} -\frac{m_1}{x^2}\left(1+xt_1\right)-\frac{m_2}{x}\left(\frac{t_1^{\,2}}{2}-\frac{1}{x^2}\right) \\ +\left(\frac{m_1+m_2t}{x^2}-\frac{m_2}{x^3}\right)e^{xt_1} \end{bmatrix}+\left[\left(\eta_2+\Delta_8\right)+y\left(d_c+\Delta_4\right)\right]$$

$$\left.\begin{bmatrix} \frac{W_O}{y}\left(1-e^{-yt_1}\right)-\frac{m_1}{y^2}\left(1+yt_2\right) \\ -\frac{m_2}{y}\left(\frac{t_2^{\,2}}{2}+t_1t_2+\frac{t_1}{y}-\frac{1}{y^2}\right)+\left(\frac{m_1}{y}+\frac{m_2T}{y}-\frac{m_2}{y^2}\right)e^{yt_2} \end{bmatrix}\right]\Big\} \quad (16)$$

Now the total fuzzy inventory cost $d\left(\tilde{TIC}\right)$ using signed distance method is

$$d\left(\tilde{TIC}\right)=\frac{1}{4T}\Big\{\left(d_o-\Delta_1\right)+\left[\left(\Delta_6-\Delta_5\right)+x\left(\Delta_4-\Delta_3\right)\right]$$

$$\begin{bmatrix} -\frac{m_1}{x^2}\left(1+xt_1\right)-\frac{m_2}{x}\left(\frac{t_1^{\,2}}{2}-\frac{1}{x^2}\right) \\ +\left(\frac{m_1+m_2t}{x^2}-\frac{m_2}{x^3}\right)e^{xt_1} \end{bmatrix}+\left[\left(\Delta_8-\Delta_7\right)+y\left(\Delta_4-\Delta_3\right)\right]$$

$$\left.\begin{bmatrix} \frac{W_O}{y}\left(1-e^{-yt_1}\right)-\frac{m_1}{y^2}\left(1+yt_2\right) \\ -\frac{m_2}{y}\left(\frac{t_2^{\,2}}{2}+t_1t_2+\frac{t_1}{y}-\frac{1}{y^2}\right)+\left(\frac{m_1}{y}+\frac{m_2T}{y}-\frac{m_2}{y^2}\right)e^{yt_2} \end{bmatrix}\right]\Big\} \quad (17)$$

5. Numerical result

The optimization techniques have the ability to compute the highly non-linear equation for the better result of any system. To demonstrate the better model, we used two numerical examples which one for crisp and the second for fuzzy.

5.1. Illustrative Examples

In this section, we consider two numerical examples for validating and the relevant importance of the developed model for sessional products.

Example 1. (For Crisp model): Consider an inventory model as per the direction and notation defined in Section 3. The following are the parametric values associated with the model:

d_o=2000, $d_c=0.5$, m_1 = 10,000, m_2 = 100, $\eta_1=0.75, \eta_2=0.4$ x = 0.06, y =.07.

By utilizing all these values, we compute the total inventory cost by using Equation (13) with the varying value of the parameter t_1 and t_2. Since the cost of the inventory depends on the parameters t_1 , t_2 and hence an investigation has been by varying it. The following are the results obtained from the analysis:

1) Initially, we fix the parameter t_2 and varying the values of t_1 from 74 to 90 and hence the corresponding variation in the TIC is shown in Figure 2. From this analysis, we can analyse the impact of the individual parameter t_1 and conclude that with the increase of the t_1, the TIC decreases.
2) On the other hand, we also analyse the impact of t_2 by fixing the value of t_1(at 74) to the TIC. For it, we vary t_2 from 5 to 26 and hence their corresponding impact on the TIC is summarized in Figure 3. Again, it is seen from the figure that with the increase of the parameter t_2, the TIC is also increases initially and then decreases.
3) Finally, the simultaneously impact of the parameters t_1 and t_2 on the TIC are computed by varying t_1 on $y-$axis and t_2 on $x-$ axis. The corresponding changes in the TIC of the system is shown on the $z-$axis of the Figure 3. It is also seen from this graph the concavity nature of the objective function TIC. The optimal solution of the developed crisp model is obtained as t1*=74.842, t2*=22.572, WO*=11434853, TIC*=1461588.

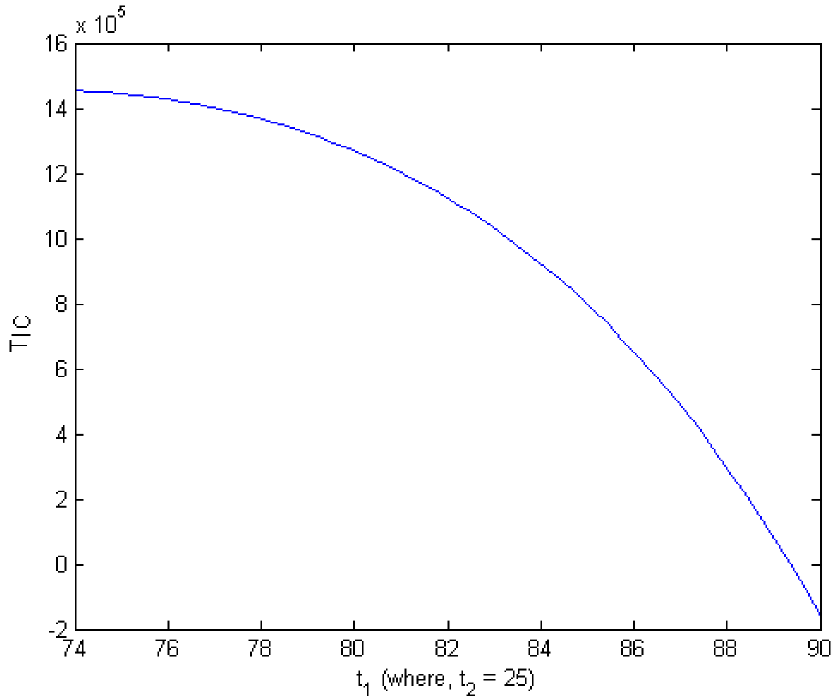

Figure 2. Variation of the Total Inventory cost (TIC) with t_1

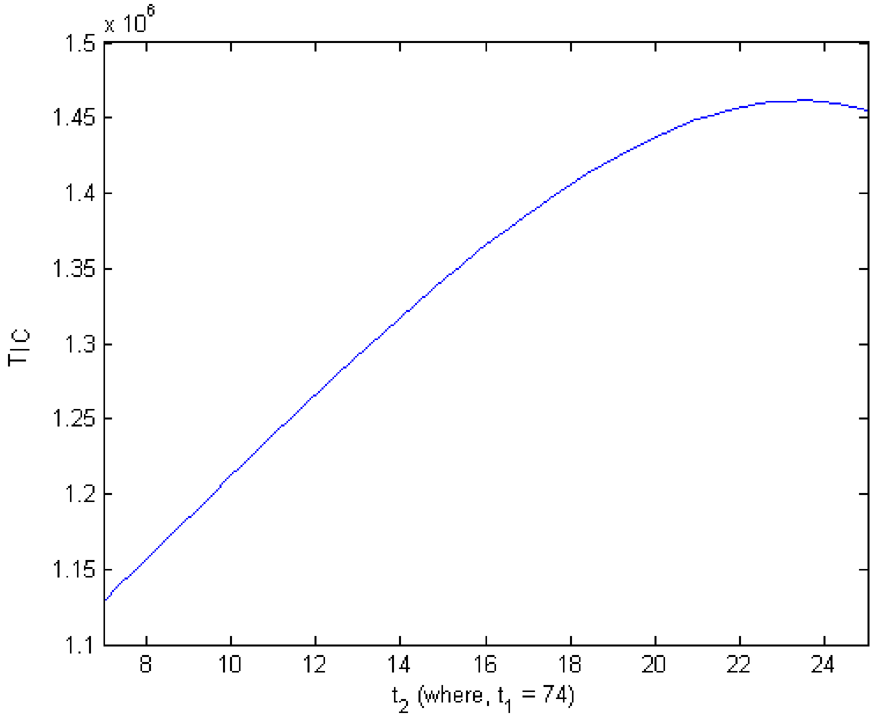

Figure 3. Variation of the Total Inventory cost (TIC) with t_2

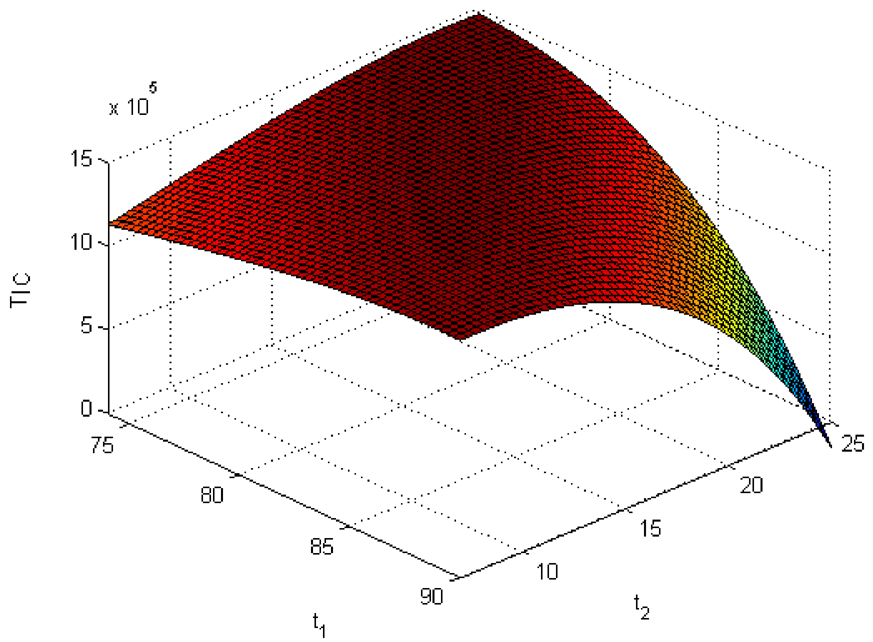

Figure 4. Variation of the Total Inventory cost (TIC) with t_1 and t_2

Example 2. (For Fuzzy model): Consider an inventory model as per the direction and symbols used in Section 4. The following are the parametric values associated with the model:

d_o=2000, $d_c = 0.5$, m_1 = 10,000, m_2 = 100, $\eta_1 = 0.75, \eta_2 = 0.4$ x = 0.06, y =0.07.

By taking such values in the Equation (18), we analyze performance of the total inventory cost under signed distance (dTIC) with the parameters t_1 and t_2. To this, we fix the value of t_2 at 25 and varies the values of $t_1$from 74 to 90 and hence their corresponding variation in the expression of dTIC is computed and the result is summarized in Figure 5. Similarly, to analyze the impact of the parameter t_2 on dTIC, we fix the value of t_1 as 74 and hence their corresponding variation is shown in Figure 6. Furthermore, both the parameter t_1 and t_2 also simultaneously effect on the dTIC as mentioned in Eq. (18) and hence we vary t_1 on $y-$axis and t_2 on $x-$ axis and their impact on dTIC is analyzed through the surface plot shown in Figure 7. The optimal solution of the developed fuzzy model is obtained as t1*=75.087, t2*=22.323, WO*=11317670, dTIC*=1454650.

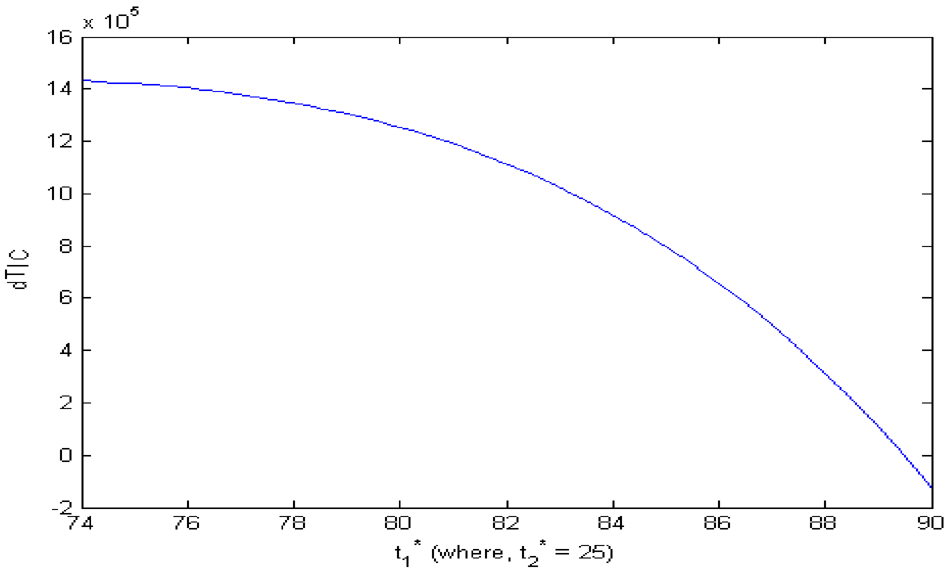

Figure 5. Variation in the Total Fuzzy Inventory cost (dTIC) with t_1

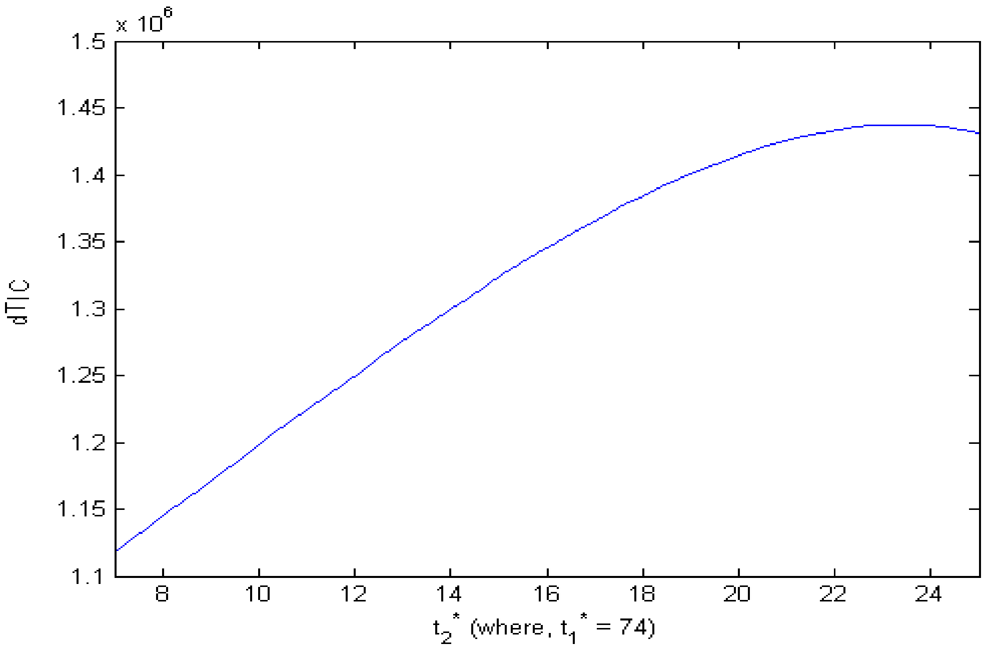

Figure 6. Variation in the Total Fuzzy Inventory cost (dTIC) with $\boldsymbol{t_2}$

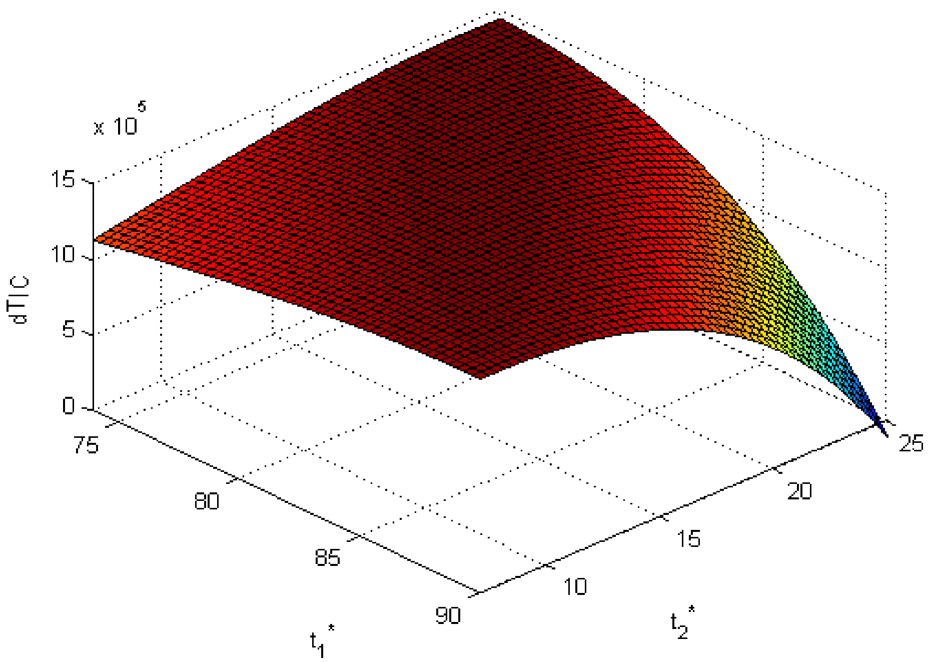

Figure 7. Simultaneously Impact of $\boldsymbol{t_1}$ and $\boldsymbol{t_2}$ on to the Total Fuzzy Inventory cost (dTIC)

5.2. Results and Discussion

To explore the effects of the optimum solution through variations of inventory constraints for the numerical values. Assumption of deterioration and linear demand in inventory management system makes additional genuine and it helpful for the business organization to decide how to minimize the total inventory cost for any company. Optimization techniques are used for solving the above highly non-linear equations for finding the optimum result for the objective function. From the above numerical examples, determination the following deductions:

- After seeing the above examples, we see that the total inventory cost function is minimum with a fuzzy environment.
- It is noticed that in the above numerical examples the total inventory cost functions are increases as increase the ordering, storage, and deteriorations costs in both the crisp and fuzzy environments.
- In Figures 2 and 3, the result is showing in a crisp environment, the total inventory cost TIC compared with the time interval t_t and t_2 for the proposed model. On the other hand, Figure 4 demonstrate the concavity of the graph between the total inventory cost TIC compared with the time interval t_t and $t_{2.}$
- In Figures 5 and 6, the result is showing in a fuzzy environment, the total inventory cost dTIC compared with the time interval t_t and t_2 for the proposed model. On the other hand, Figure 7 demonstrate the concavity of the graph between the total inventory cost dTIC compared with the time interval t_t and $t_{2.}$

6. Conclusion

In this paper, an improved inventory model has been described under two warehouses situation – own and rented with fuzzy constraints. In our day-today life, especially for the sessional items, the price control of any business organization is a big challenge; such as food, cosmetic, and medicine products are deteriorated after the finished products. It is observed that the sessional period of any product is sensitive to the price changes due to the increase and decrease of its ordering cost, storage cost, and deterioration cost of the product. In this work, it is supposed that the demand is a linear function of time and different deteriorating in both warehouses. The objective of the work is to minimize the total inventory cost which includes ordering, storage and deterioration cost for both own and rented warehouses. A mathematical derivation for such cost under the crisp environment is derived. Further, to handle the uncertainties in the various parameters, we formulate the fuzzy inventory model under the uncertain environment. For this, we utilized the signed distance and the triangular fuzzy number to express the uncertainties in the model. The stated models have been demonstrated through numerical examples under the both fuzzy and crisp environment. The impact of the parameters t_1, t_2 on the total inventory cost are also analyzed through the surface plots. The decision-maker may analyze the behavior of the system through such graphs and hence can choose the optimal choice for t_1 and t_2. The concavity nature of the considered model is also demonstrated. In the future, we will elaborate the stated model to make some advancement, such as quadratic demand, stock-dependent demand, variable deterioration rate, maximum lifetime, shortages, inflation and trade credit policies, etc.

Conflicts of Interest

The authors declare that there is no conflict of interest.

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